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Real Options and Bandwidth Markets: Dark Fiber Valuation

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Abstract

Bandwidth markets start to make possible a real options analysis of dark fiber value by providing market price information for a related determinant of their value: the value of transmission either through lit fiber on the same route and/or related routes. Whilst a real options analysis of dark fiber may be forward looking with respect to bandwidth markets, the market for dark fiber itself is active, especially in the metro segment. Also, given the indefeasible rights of use run for 20 to 30 years we are not starting too soon. We present a valuation methodology that takes into account the salient characteristics of dark fiber itself and of the stochastic spot price process for bandwidth. We include the decreasing technology cost with time and how this changes when different lighting capacities are considered. We treat the spot price of bandwidth as decreasing and mean reverting. Critically we model the effect of adding additional lit capacity on the bandwidth market: prices drop. We also consider the value of the investment given a pre-existing investment on the same route by the buyer of the dark fiber. We examine the multiple lighting factor, that is, lighting equipment can be replaced later. We find that lighting cost, spot price volatility and the strength of the mean reversion are all important. Demand elasticity is the least significant factor whereas relative capacity added both with respect to existing capacity and to the buyers previous position are relevant over certain ranges.

1 Introduction

This chapter describes applications of real options to bandwidth markets, specifically to the valuation of dark fiber. Bandwidth markets start to make possible such an analysis by providing market price information for a related determinant of their value: the value of transmission either through lit fiber on the same route and/or related routes. However, bandwidth is not the only commodity expected to emerge in a world where the outsourcing trend shows no signs of slackening — even in the bear market conditions after the dotcom meltdown of late 2000. Storage and servers, despite the complexity of the latter, are also expected to become commodities. The natural place for these is, of course, carrier hotels which have QoS-guaranteed network connections, connections to metro fiber rings, and are large enough to have a varied set of internal customers for commodity resources. Thus the scope of real options applications can be expected to increase in this developing world of online utilities.

Real options applications in online utilities is forward looking in that bandwidth is expected to fully commoditize only over the next three years (2002-2005) and the first on-line public storage and server markets have not yet been observed. We note that EnronOnline went live with firm point-to-point bandwidth prices in February 2001 and there is a flurry of startup and venture capital activity in the on-line server and storage market areas. Given that indefeasible rights of use (IRU's) are in the 20 to 30 year range we are not starting too soon. IRU's may also be counted as assets by investors or company analysts.

Whilst a real options analysis of dark fiber may be forward looking the market for dark fiber itself is active, especially in the metro segment. However, justification for present valuations appears to be constructed on a case by case basis. With recent changes in the telecom market outlook there may be interest in more standardized approaches.

The idea of real options is to value a physical and non-traded asset relative to the decision possibilities the asset contains and relative to assets, with which there is some relationship, that are traded on a market. One objective of such a valuation is to obtain a value for an asset that is consistent with how a market would value that asset. A second objective is to identify the best decisions for operating that asset from a financial point of view taking into account the limitations that the physical asset, and management policy, impose.

A good technical introduction to real options is [DP94] more general audience works are [AK99] and [Tri99] whilst [CA01] provides a hands on approach. [AN99] provides many supporting arguments for the application of real options to telecommunications economics and is more qualitative. Probably the best single case study is that of Lund ([Lun97]) whose doctoral thesis on offshore oilfield development has the practical details and exposure of underlying reasoning and alternative approaches not typically available in other publication types.

Dark fiber is a step in the transmission capability ladder which starts with a right-of-way and continues through lighting changes and fiber updates (new pulls). Each step on this transmission capability ladder is a costly and, to a large extent irreversible, decision to be taken in the face of demand, price and rate of technological improvement uncertainty. A real options analysis aims to value the fiber today by identifying the optimal future decisions under all possible realizations of the future uncertainty.

The rest of this Chapter is organized as follows: Section 2 introduces real options, describes where they have succeeded and where they have failed; Section 3 introduces the business setting of dark fiber and previous related work; Section 4 describes our real options valuation model for dark fiber and discusses parameters and their values; Section 5 describes the valuation results and the model behavior; Section 6 concludes.

2 Introduction to Real Options

2.1 What is a real option?

A real option is a decision possibility with respect to a physical and non-traded asset. A financial option is a decision possibility embodied in a traded instrument such as a call or put option on a stock or commodity future (quite arbitrary complexity is possible in the design of these decision possibilities). The idea of a valuation of a physical asset using real options is to value the physical asset as the combined sum of its decision possibilities relative to some related market or markets. The aim is to obtain a market valuation for a non-market asset. The closer the related markets are to the actual physical asset the more objective the valuation will be and hence the more acceptable to different parties (e.g. the buyer and seller of the asset or the its tax appraiser).

The amount of effort to put in to a real options analysis of an asset depends on the intended use of the results and the quality and availability of the data. For example, to run a gas-fired electricity generator optimally a high level of operation detail needs to be included together with electricity and gas market details because the outputs are the day-to-day operating decisions. In this case all the required data is available at the necessary level of detail. To buy or sell the same generator a much lower level of detail is needed — the objective is only to obtain a value that is within some specified percentage of the value for the generator's optimal operation. Alternative valuation methods, such as cost, multiples (where applicable), or other recent sales provide none of the objectivity or detailed decision support possible with real options.

A clear understanding of the physical asset details is required input to any real options analysis.

2.2 Where have real options succeeded?

The three classical examples of real options applications are oil fields, gas storage, and gas-fired electricity generators. In these cases the assets themselves are not actively traded on a market but important determinants of their value are traded. Crude oil spot contracts, futures and options on futures are all traded. Both gas and electricity are also traded. It is not true that in these cases markets can be used to remove all subjectivity from valuation or operational decisions. Often there are significant value determinants that are not traded and thus must be left to the subjective opinion of the experts or managers. However, markets can be used to significantly reduce uncertainties about key determinants of the value of these assets. This is the essence of a real options analysis: reducing the number of subjective inputs. Even in financial markets option pricing under Black-Scholes there is a free parameter, the volatility, which must be forecast in some way.

There is little commercial software available for using real options to value oilfields. The main reasons are: diversity of oilfield types; importance of detailed geological and test information; length of life – 20–30 years; and finally the high capital investment involved making customized analyses attractive. In the case of gas storage and generation assets, these are sufficiently uniform, and relatively inexpensive (compared to an oilfield) to permit the construction of commercial software. This availability highlights the success of the method, and its relative novelty.

2.2.1 Oilfields and Gas Storage

Real options are a natural analysis method for oilfields ([SM99], [Lun97]) and gas-storage systems (natural and artificial reservoirs). There is a clear market for the product which is itself a commodity.

The three biggest determinants of oil-field value are oil price, reservoir volume (or possible production profile), and technological change[1]. The first can be valued with respect to the market for crude oil (hedged) the second two cannot - no traded assets are correlated with an underground reservoir production profile or with the rate of improvement of extraction technology. However even being able to hedge one of the value determinants is valuable additional information.

For undeveloped fields - and even more so for exploration licenses - the unhdegable uncetainty is very large with respect to the reservoir volume and production profile. For mature fields this is greatly reduced although new technologies continue to improve the recovery fraction. However removing price uncertainty - or at least taking the market view of it - was an enormous improvement over previous methods. Previous view of future commoditiy prices were usually a small number (three) deterministic scenarios or simply a break-even price for the reservoir ignoring all development flexibility. The break-even method, even when flexibility is included, does not enable valuation and hence comparison or sale/securitization of assets. Thus for these applications real options were a natural fit and provide significant value.

Gas storage reservoirs, often exhausted oilfields, exist to perform time-arbitrage. Gas prices are highly seasonal so it is profitable to be able to buy cheap gas in summer and then sell it at higher prices in summer. Note that gas, unlike oil, has high transportation costs so it is common to see quite different gas prices in different locations. Transportable storage (i.e. liquified natural gas carriers – LNG) is expensive. This time-arbitrage that gas storage profits from is reduced by their actions but not removed because, again, cheap enough storage is limited. Thus a seasonal component to gas price remains.

How do you operate a gas storage reservoir to maximize profits? This is a classic real options problem. This combines a management policy with physical limitations of the storage reservoir and the gas market. The management policy for example might be to have the same amount of gas in the reservoir every 12 months from, say, May. The physical limitations of the reservoir are its capacity together with how fast gas can be pumped in or out. The gas market is now online and sophisticated in terms of (financial) products available. The value can also be changed by creating access to different gas supply networks.

In this area academic literature appears to lag industry practice with only a few recent conference presentations whilst commercial software products have also recently appeared (e.g. Storage Valuation Advisor from Lukens Energy Group, GasOptions from Caminus, or other products from FEA [2]) implying earlier development.

2.2.2 Gas-Fired Electricity Generators

Valuation of gas-fired electricity generators [TB99, DJS98] is another clear success for real options. Here both the input (gas) and output (electricity) have markets and the optionality,

[1]Technological change has not yet been included in any real options literature on oilfield valuation despite its importance — every oilfield engineer can give a list of developed/improved fields where technological change radically improved profitability

[2]No explicit or implied recommendation either by the author or the authors' employer should be attached to these references.

i.e. being able to decide when to switch the generator on/off, is critical. These generators only operate profitably when the spark spread (difference between electricity and gas prices) is sufficiently large. Since electricity price volatility can be very high, speed of switching power production on and off is important. Annualized volatility of spot and (month ahead) futures prices can be upwards of 100% in regions where there is no significant amount of storage (i.e. no hydro or pumped storage). Gas turbines have switching times measured in minutes unlike thermal or nuclear plants which take days to switch. Thus they are suited to use as peaking plants. Although gas-fired generators do switch much faster than thermal generators there is still a maintenance cost for the physical switching as well as personnel and other costs which must be included in a valuation or operation calculation.

This is probably the cleanest and most dramatic example of the methods' success. In fact commercial application was arguably ahead of the academic literature (e.g. see Enron's 1999 Annual Report). Again commercial products have appeared to value generation assets from some of the same companies offering gas storage valuation products.

2.2.3 Specialized Applications

Lastly we mention four more specialized applications where new quantitative techniques or adaptations were used. Research and development valuation at Philips electronics [LP99]; offshore rig lease option valuation at Schlumberger Sedco Forex (now Transocean Sedco Forex) [KT99], [KT01]; drug development at Glaxo Welcome [Mic99]; and airplane option valuation at Airbus [Sto99]. In each of these cases significant amounts of market information were available and used in projects which had direct impact on the companies involved.

2.3 Where have real options failed?

Where no market information is available or where the market setting of a business or asset is ignored, any approach calling itself real options has, from the point of view of a real options analysis, failed. The defining characteristic of the approach is a link to a market in an attempt to obtain a more objective valuation. We offer the following list of indications that the real options method has failed in its application.

1. A decision tree approach simply re-branded with an options language: this fails to meet the standard of attempted objectivity in valuation. A simple test is the question "was market information used?". Without a related market valuations return to being subjective. This is not bad it just reflects the reality of many valuation exercises that little or no market information is available.

2. Mathematical finance machinery is employed without modification (e.g. Black-Scholes option pricing formulae) to impart some aura of objectivity to a series of subjective exercises in parameter estimation.

3. Internet start-up valuations called real options valuations where all that is being done is a decision tree with the same subjective "good" / "medium" / "bad" list of scenarios that real options in oilfields was a successful alternative to. Objectivity is not created by a change in language.

Note that these are the authors' views and some will certainly hold the view that the real options language is more appropriate and fashionable than that of decision analysis and thus a re-branding of decision trees as real options is timely. Certainly more dissemination of structured problem framing is good for decision makers but they should be wary of claims of objectivity or indeed of the numbers themselves that complex methods produce when not objectively justified. Caveat emptor.

2.4 Asset Value

Valuation using real options is actually rather simple to describe mathematically. The whole art is in fitting the valuation structure to the details of the physical asset's decision structure and constraints, and to the relevant markets and financial instruments.

The actual value of an asset also depends on the business climate and the business assumptions that are made. This is a much broader topic. In the section on dark fiber (Section 3) we will go into the business assumptions for both in detail. Business settings clearly depend on when they are written. In time certain hypotheses are validated or falsified and more hypotheses about the future are generated. We are writing in mid 2001 but attempt a certain robustness. The reader must exercise judgement on which parts of our hypotheses are more or less relevant to their particular situations.

Definition 1 *The value of a physical and non-traded asset, $\mathcal{A}$ is the maximum obtained relative to all possible future decisions ($\mathcal{D}$) which include taking positions in financial market instruments ($\mathcal{F}$), under the constraints that management policy imposes ($\mathcal{M}$), and under the physical constraints ($\mathcal{P}$) inherent in the asset. We call this value $\mathcal{RO}(\mathcal{A}, \mathcal{F}, \mathcal{D}, \mathcal{M}, \mathcal{P})$ the real options value of the asset, or $\mathcal{RO}(\mathcal{A})$ for short.*

This definition basically provides the valuation recipe as an optimization problem over possible decisions and subject to constraints from different sources. The market instruments will generally include US or UK government bonds which are regarded as risk-free hence there is a built in comparison with the riskless rate of return. A poor investment possibility will have as the best decision not to take the project at all but to invest in bonds! The definition also specifies the pieces that go to make up a valuation. For example a project may be constrained to positive market positions (i.e. no short selling), or positive cashflow every quarter after the first year.

3 Dark Fiber

Dark fiber is unlit optical fiber. This may be in long-haul networks or in metro rings. Fiber may be available between public access points or pooling points. Alternatively it may simply be along some right-of-way owned by the company laying it.

3.1 Previous Work

Direct valuation of options on commodity bandwidth, taking the existence of the network explicitly into account, has been studied by [Kep01, KC01a]. These, however, do not take into account all the factors required for a valuation of dark fiber. Firstly those options are options on a fixed capacity for a fixed length of time. Dark fiber is of variable capacity depending on lighting and this is of variable duration depending on various cost factors. Also these works do not model American style exercise, the changing exercise price for lighting, or the compound nature of the lighting decisions. They both make the point that the existence of the network changes the underlying price structure dynamics upon which option prices are based. The closest other work is in two areas, portfolio liquidation, and capacity upgrading.

Portfolio liquidation (e.g. [AC01, BHL99, SH00, HY00]) looks for the optimal liquidation (sell-off) strategy for a portfolio in a given length of time. Dark fiber is similar to an option on such a situation: when the fiber is lit the capacity can be sold. The significant factor in these models is that selling off the portfolio changes the price. This is usually factored into two pieces, an immediate temporary factor and a permanent impact factor. A number of papers in this area have explored empirical and theoretical shapes of the price impact function. Transaction costs are sometimes included in this setting.

Capacity upgrading is the general case of dark fiber lighting but, so far, has only been tackled in a single provider setting. The most recent work, [dFV01], takes demand as increasing stochastically and exponentially with price decreasing deterministically and exponentially. Price per unit of capacity is also taken as decreasing with increase in installed capacity size (economies of scale). They then formulate the option value and best upgrade time as a function of observed demand level. Unlike the portfolio liquidation case there is no affect on price from the increase in available capacity — it is just that you can only sell up to the current (stochastic) demand. Upgrade costs decreased at the same rate as the deterministic price decrease. Here we deal with the valuation of dark fiber taking prices to be stochastic, mean-reverting and decreasing. We additionally assume that the market reacts to new capacity: prices drop. However, this drop is of limited duration, the price process is mean reverting. The valuation here is from a market point of view rather than the value a single provider can extract from new capacity where the provider can set the price.

3.2 Simple Valuation Arguments

Here we go through the basics of some valuation arguments and describe their connection with dark fiber.

If we assume the existence of a forward price curve for bandwidth then we have access to a trivial valuation strategy: for all future times calculate the net present value of the capacity made available by lighting given the x year lifespan and the lighting equipment cost. Given this basis, after the fiber is lit there is no timing problem: sell everything as soon as possible because the forward curve goes down — if you wait the value decreases.

On the basis of the above argument one could conclude that dark fiber is valueless. The reasoning goes as follows (assuming lighting costs decrease to the same degree as prices): if the fiber is not now lit so the cost must be too high; the price of bandwidth is falling so the revenue will be less the longer the lighting is delayed and lighting is not becoming proportionately cheaper. Hence if fiber is not now valuable it never will be.

This trivial valuation is optimistic because new capacity impacts the actual forward curve (downwards). It is misleading because the forward price is *not* the expected future spot price, there are utility and risk factors built in. It is also misleading because it does not take into account the balance between supply and demand. It is pessimistic because it does not take into account any increase of demand either because of price elasticity of demand or otherwise.

However this back of the envelope argument does imply that future lighting costs must drop quickly or demand must increase if dark fiber is ever to have positive value. Lighting cost per unit of capacity can drop either because the technology becomes cheaper and/or because of economies of scale.

3.3 Business Setting

Investment There have been extraordinary capital investments and network build-outs in the past 2–3 years since 1998 with many new entrants, even very large ones, e.g. Williams Communications, a $US10B spin-off from Williams Energy, Enron Broadband, etc. This implies that there is already lit fiber from at least two and probably several suppliers on every significant route where there is dark fiber. This is true even for the new under-sea cables which start with only a very small proportion of fiber lit, e.g. 10Gb of, say, 3.2Tb final capacity. This lit fiber is typically pre-sold to several carrier or carriers' carriers. Thus we expect market-based spot prices to be available for most dark fiber routes.

Forward prices We will also assume the existence of forward prices. Short duration forward contracts are typically constructed from a mix of long term contracts and financial swaps. Downward sloping forward curves (i.e. cheaper later) imply capacity is either being added to the link in question or that capacity is being added to other routes which in combination can be used to create an equivalent QoS alternative. The forward curve is the markets appreciation of the speed with which capacity will be added to the market together with the resultant increase in demand from price elasticity. However, the market makers are aware that owners of dark fiber have many options on lighing so they will include a risk premium for this uncontrolled factor in their quotes.

Supply If all the existing dark fiber were lit prices would generally collapse (as has been seen on routes with oversupply, e.g. London-New York). However this may or may not be economic depending on price elasticity of demand. Whatever the case, in general, lighting a fiber may affect prices. Certainly where this adds significantly to capacity this will be true and this is typically the case at present. The closest similar situation in the financial markets is when an investor wishes to unload a large, say, stock position. This will both increase the supply of the stock and send a signal to the market about the opinion of a large player. In the case of bandwidth the signal is in the opposite sense. Thus models have been developed by institutions to give the optimal sell-off (liquidation) strategy, e.g. [AC01]. This is very similar, in concept, to lighting dark fiber except that lighting is an option whereas the sell-off models take the length of time to sell-off as a given. However, as stated above, the market will factor in to the forward curve an expected rate of addition from all participants.

Demand The most general demand variable is the price elasticity of demand. What new applications will cheaper bandwidth make economic? Additionally, what new applications will bandwidth availability make possible? Bandwidth availability needs to be broken down into three categories: backbone; metro; last-mile. A retail application (e.g. online gaming) requires at least the last two types of bandwidth whereas a large enterprise application without a retail component needs only the first two. Currently there is little evidence of price competition on the last mile or any hint of oversupply. It is often argued that the last mile is the major current bottleneck on unleashing demand for bandwidth. This is certainly true on the retail level. However enterprise strategy is starting to favor outsourcing and carrier hotels which (not so) coincidentally also removes the last mile cost and availability problems.

Competition We assume that the owner of the dark fiber is not in a monopoly position in that there are other firms that are capable in adding to transport capacity between the same endpoints and at equivalent QoS. Even if the owner has monopoly power on the link itself this may not prevent the construction of alternatives using several links. It is difficult to maintain even local monopoly control of a network commodity.

Timing Fiber cannot be lit instantaneously. When lighting manufacturers have spare capacity there will be a lead time of weeks to months. However when there are supply shortages, as happened with optical fiber in late 2000, the delay in obtaining equipment may go up to a year or more for new customers.

Liquidation Lit capacity needs to be sold. Lighting is a granular process, especially in terms of physical installation, whatever units of capacity increase are available. The owner has the choice of how much of the new capacity to make available to the market and when to make it available. We assume a non-monopoly, and commodity, situation thus the owner of the dark fiber can observe the price and forward impacts that all participants produce by lighting fiber.

4 Valuation Model

We develop here a valuation model for dark fiber, i.e. unlit optical fiber with DWDM capability. The objective of this model is to make it clear how the value of the dark fiber depends on a combination of economic and market factors. Thus the model will go for simplicity of exposition over sophistication. We will indicate later what sophistication are required to move this demonstration model to a level suitable for commercial use and how to do this. However the objective here is to describe clearly how a valuation can be constructed.

4.1 Basic Economics

Two stylized facts of the bandwidth market are that demand is increasing, even exponentially increasing, and that prices are falling (where there is competition), also possibly exponentially. These observations demand understanding from a basic economics point of view because usually increasing demand leads to higher prices. Clearly there is another factor in play. This factor is cost decrease: both because of economies of scale and because of improvements in technology. Technological improvements have been dramatically illustrated with the development of DWDM in the last 5 years. Economies of scale have been accessible because of significant demand elasticity. That is, if the price is reduced the amount used increases significantly. This is called an industry with external economies or a decreasing cost industry.

We will start from a constant elasticity of demand, for a fixed point in time t, model

$$p = \left(\frac{a(t)}{q}\right)^{1/e} \tag{1}$$

and a similar constant elasticity of supply model

$$p = c + d(t)q \tag{2}$$

Figure 1 shows several examples of both models. Market prices are at the intersections of the supply and demand curves. Clearly for observed prices to decrease whilst demand is increasing ($a(t)$ increasing in Equation 1) $d(t)$ must be rapidly decreasing in Equation 2.

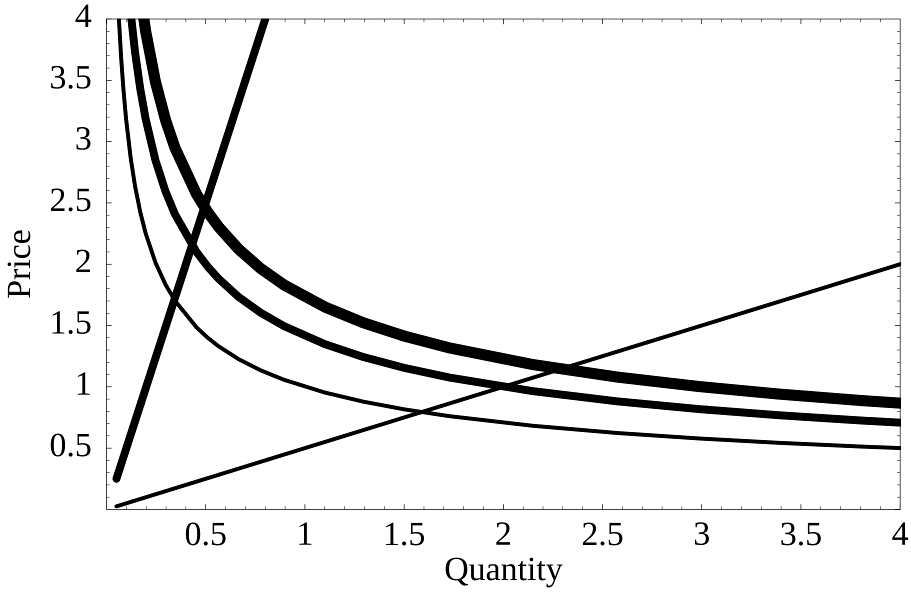

Figure 1: Constant elasticity of supply (lines) and demand (curves). Thinner lines are for greater elasticities. .

We also have, taking $c \equiv 0$ for simplicity:

$$p(t) = (d(t)a(t))^{\frac{1}{e+1}} \quad (3)$$
$$q(t) = p(t)/(d(t) \quad (4)$$

Note that Equation 3 gives the relationship between the decrease in the cost with time and the decrease of the price with time. These are not identical as has been assumed elsewhere for bandwidth (e.g. [dFV01]). By specifying appropriate functions for $a(t)$ and $d(t)$ we can solve for observed prices, see Figure 2. Clearly this would be the long term equilibrium prices $L(t)$ (which is a contradiction in a non-stationary model) if this were a stationary model. However supply and demand are also uncertain and stochastic. What we can use $L(t)$ for is the long term price trend within a stochastic price model. We will base our stochastic price model on [SS00, KC01b].

4.2 Bandwidth Spot Price Model

We use a mean reverting stochastic price process with a downward trend. We also include the effect on prices of adding new capacity to the market. This is highly important for the valuation of the real option to add capacity. Where the added capacity is a significant fraction of the market it will depress prices. These prices will recover as demand increases over time but they will only return toward the long term trend and not the previous prices.

We use the following model for the spot price of bandwidth

$$X = log(S) \quad (5)$$
$$dX = \eta(Y - X)dt + \sigma dW - \gamma dN \quad (6)$$
$$dY = -\nu dt + \rho dZ \quad (7)$$

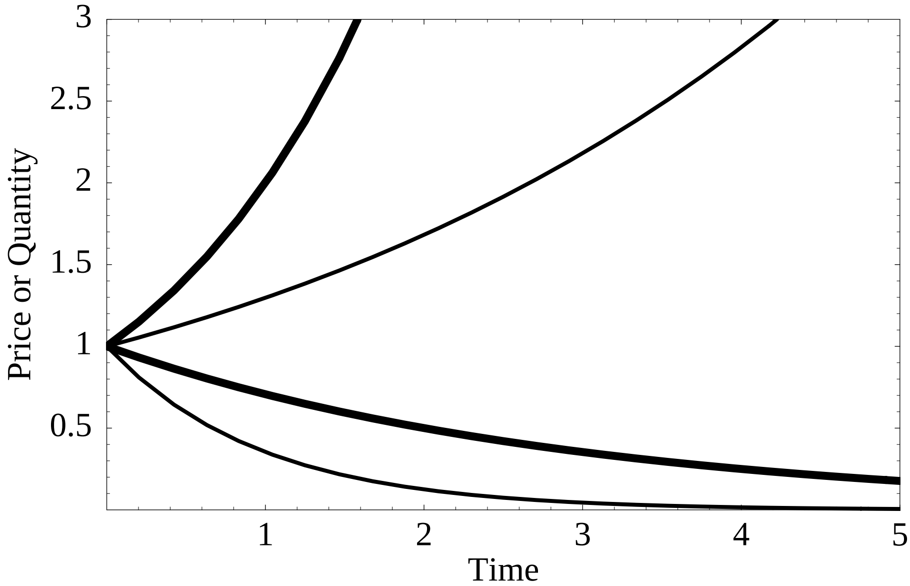

Figure 2: Observed price and quantity forecasts versus time (heavy lines) given cost and demand forecasts (light lines). Demand and supply elasticities cause the differences between the light and dark lines (see text for details). Basically higher prices can be charged than changes in technology would indicate because of increasing demand. Demand is also price elastic so for decreasing prices more quantity is actually delivered.

This is similar to [SS00] except that we are basically modeling the log of the price rather than the price directly. In this respect Equation 6 is similar to [BK91]. The whole setup has also been mentioned by [Reb96] and a more sophisticated version including network effects explicitly was used in the bandwidth context by [KC01b]. By network effects we mean that prices on one route will affect prices on other routes (and vice versa) because buyers will always pick the cheapest route (at equivalent quality of service) and thus shift demand. We are also neglecting the non-storable nature of bandwidth which would usually require the possibility of price spikes. (The nature of price spikes in a bandwidth market are covered in detail in [CK01]).

Since the costs and prices of bandwidth have, by assumption, some long term trends it is reasonable to have those incorporated into the model and these are encapsulated by Equation 7. This also provides the justification for the mean reverting nature of the observed prices in Equation 6. This mean reversion is a common feature of commodity prices. In a competitive market no single player can set prices but all are, to some extent, price takers. Thus there is uncertainty about future prices and this is incorporated via the driving Weiner processes dW and dZ. The uncertainty of the long term trend does not add significantly to the current exposition so we set $\rho = 0$ for simplicity.

The last factor to note is γdN in Equation 6. dN is not a random variable and $dN = 1$ when new capacity is added to the market. γ describes the effect on market prices of adding capacity according to Equations 1 and 2. Adding capacity reduces prices in the short term but does not affect the long term trend (Equation 7). This γdN term is a "permanent effect" in the language of [AC01] although here its effect is mitigated by the fact that the price process is mean reverting. Figure 3 shows the effect of this factor on the price forecast of a particular history after the capacity is added.

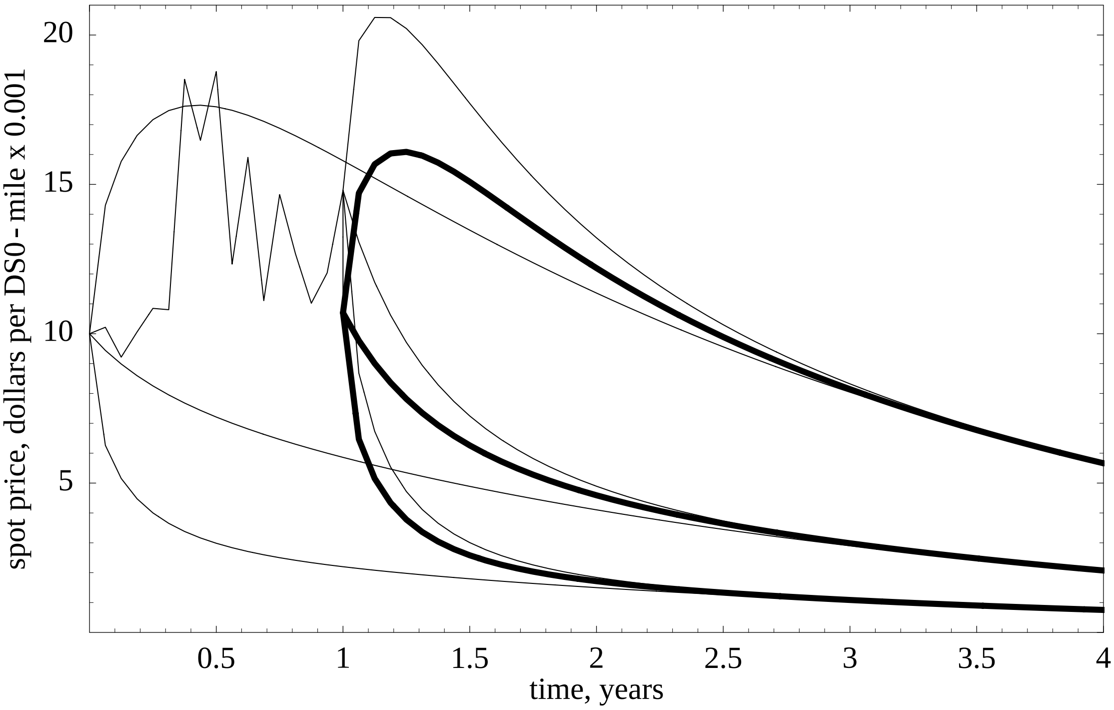

Figure 3: Confidence intervals for spot price process. Percentiles shown are 5%, 50% (median) and 95%. Later starting confidence intervals are for the future evolution of a particular price history (jagged line). Darker confidence intervals take into account the effect of adding capacity to the market and hence reducing prices.

4.3 Option Modeling

Dark fiber is an option to obtain lit fiber, at any future time up to the contract terms in the "irrevocable right of use", by paying the exercise price of the lighting equipment. This process can be repeated by successively upgrading lighting equipment. Depending on the contract terms, dark fiber can even include the option to re-blow/re-pull the fiber when its DWDM capacity is reached and when successive fiber generations are available. The choice of lighting equipment provides capacity options that are non-linearly related to each other. Thus dark fiber represents a multiple, compound, American-style, option. The exercise prices of these options are not constant but decreases with time. Finally the payoff is not only at the exercise time but is the revenue from all future use until the maintenance costs are higher than the expected future revenue (including future maintenance costs). We will now detail and model each of these factors.

4.3.1 Lighting and Maintenance Costs

Lighting costs $K(c,t)$ per unit capacity depend on capacity to go to c and time t and are only available as discrete choices, say $\{c_1, c_2, c_3\}$. Typically at the moment (mid 2001) these are OC48, OC192, OC768 for each fiber under consideration (or for each wavelength in a DWDM setup). Clearly, other choices will be relevant as time goes on but the structure of the problem will not change. We will model single fiber value rather than fiber bundles. Note that the extension to multiple fibers is non-linear. Adding twice the capacity to the market does not guarantee twice the revenue because of the non-linearity of the price drop due to price elasticity. In fact in many cases the question will be how many fibers (or wavelengths) to add not just what lighting to put there.

Higher capacities are typically used when they become relatively cheaper [Cla01]. This is also indicated by other evidence, e.g. [dFV01]. Maintenance costs $M(c,t)$ per unit of capacity as a proportion of lighting cost are also relatively cheaper for higher capacities. However, since maintenance is generally a personnel intensive activity that is not automated, these costs may not decrease after the lighting is installed. In fact they may keep pace with inflation. Note that dollar maintenance costs are $M(c,t) \times K(c,t)$ for dollar lighting costs of $K(c,t)$ per-unit-capacity. Thus we take

$$K(c,t) = K_0 e^{-kt} e^{-lc} \tag{8}$$

$$M_0(c,t) = M_{00}/c \tag{9}$$

$$M(c,t) = M_0(c_i,t_i) e^{(r-\delta)t} \tag{10}$$

Equation (8) says that lighting costs decrease independently with time and capacity. A joint function could also be used, this was chosen for simplicity. Equation (9) says that maintenance costs are a fixed percentage of each installation: e.g. lighting with twice the capacity has half the per-unit-capacity maintenance charges. This could be changed as required. Equation (10) simply states that, after installation of capacity c_i at time t_i, the maintenance charges for that installation increase with inflation as some delta, δ, below the riskless interest rate, r.

4.3.2 Option Payoff

Unlike the payoff from a typical financial option the payoff from lighting dark fiber lasts until the expected future revenue is no longer positive. Whenever the lighting decision can be exercised it is possible to calculate the expected revenue using a (simple) stochastic dynamic program. The first stage is to construct a spot price tree from the exercise point. This will start from $S(t_i)/\gamma(c,C)$ (i stands for installation) where the current pre-lighting price is $S(t_i)$. The spot price, in the long term, is decreasing and mean reverting so at some future point there is no spot price that is above the maintenance cost. We could work backwards from that time using the following relationship:

$$R(t) = \max((S(t) - cM(c,t)K(c,t_i)) + e^{-r\Delta t} E_t[R(t+\Delta t)]\Delta t, 0) \tag{11}$$

In fact what we do is simply go out a fixed number of years and work backwards as specified. Providing this period is sufficiently long the error tends to zero and we test this. The $S(t)$ in Equation (11) is the obviously the post-exercise spot price process. The expectation is taken with respect to the current state of knowledge at t at each time step moving backwards to the lighting installation time t_i. The total payoff from lighting installation is $R(t_i)$ and the option exercise price is $K(c_i,t_i)$

$R(t_i)$ must be calculated independently for each node on the spot price tree and for each different possible lighting decision because the new available capacity changes the future price development and this is different depending on the spot price at exercise. Equation 11 can easily generalize to take into account many different lighting possibilities.

Note that $\gamma(c,C)$ is defined from Equations (4) and 3) under an increase c of capacity starting from a capacity of C and is given explicitly later (Equation 19).

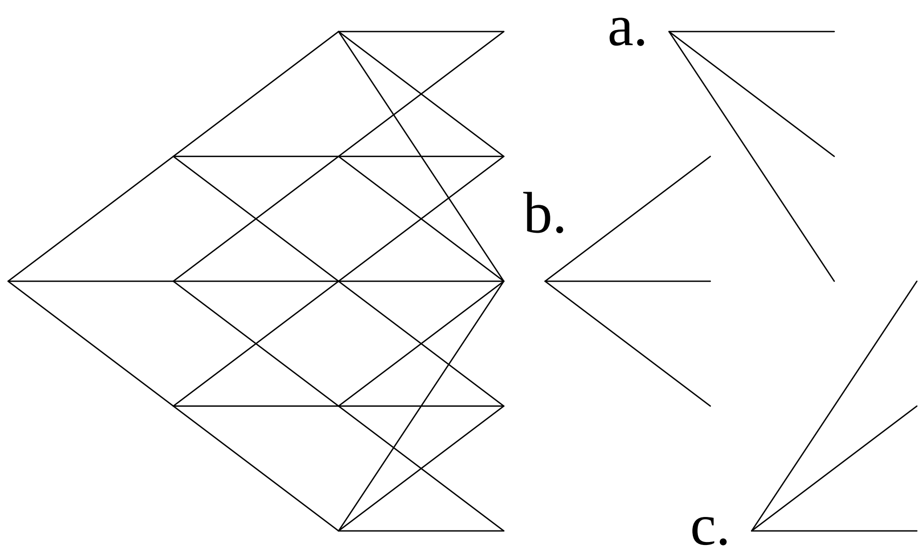

Figure 4: Basic trinomial tree parts. Tree spread is limited to match mean reversion characteristics of price process. Branching methods used at different part of the tree are shown in (a), (b) and (c). Probability of top branch is p_u, of middle branch p_m and of lowest branch p_d.

4.4 Numerical Methods

To calculate option values, $R(0)$ from equation 11, we must evaluate the recursive relationship (or Bellman equation) for $R(t)$ bearing in mind that the spot price process $S(t)$ is changed by option exercise. We use a tree method whereby we discretize $S(t)$ over time and over possible values at any time. In fact, at each option exercise we must construct a new tree starting at that point taking into account the change in $S(t)$. This is because option exercise adds capacity to the market and this changes current and following underlying prices. This is a unique numerical feature of this work. This means that the final tree is semi-bushy in that tree branches re-combine except across option exercises. Re-combination means that an up movement of S(t) followed by a down movement of produces the same effect on S(t) as down then up. Once we have this semi-bushy tree we can simply start at the end and move recursively backwards applying 11 at each point. (Note that direct simulation for valuation is not applicable here because we have an American-style situation — option exercise can take place at any time.)

No tree construction available in the literature is directly applicable. The construction methods available for the stochastic process we use assume that the process being modeled is an interest rate. This means that for calibration the values on the tree are compounded in order to match a given calibration (yield) curve. Since we do not have compounding we need to provide a new calibration method and this is given below. Our method is based on [Hul00] (Chapter 21, sections 11 and 12).

4.4.1 Trinomial Tree Basics

Figure (4) shows the basic character of the trinomial tree. Spread is limited so as to match mean reversion characteristics of price process. This entails changing the branching method at the boundaries (as in (b) and (c) above) when a sufficient spread is attained.

4.4.2 First Step

The first stage of the tree building for Equation 6 is almost identical to [Hul00]. We build a tree for

$$dS^* = -\eta S^* dt + \sigma dW \tag{12}$$

where we use S^* to indicate the dummy variable for which we are building the first stage. The values for $\Delta S = \sigma\sqrt{3\Delta t}$ and the probabilities for the tree branching (p_u, p_m, p_d) are also the same (see Appendix A). The difference comes at the second stage.

Let

$$\alpha(t) = X(t) - S^*(t) \tag{13}$$

Since we have

$$dX(t) = (\eta Y(t) - \eta X(t))dt + \sigma dW \tag{14}$$

it follows that

$$d\alpha(t) = (\eta Y(t) - \eta\alpha(t))dt \tag{15}$$

which, for any given functional form of $Y(t)$ we can integrate numerically at the same granularity as Equation 6. As mentioned in [Hul00] this does not exactly match the term structure and so we employ the scheme below. Where we do use this is after a price drop due to the addition of capacity to the market. We then require a method to match $\alpha(t)$ back to the previous $Y(t)$. This method is used up to the point where the two curves intersect. After this point the previous $Y(t)$ is used.

We can now also calibrate $Y(t)$ to a given forward curve, $F(t)$, or equivalently to any other future price prediction as desired. Here we use the expected price predictions that we derive from supply and demand. This is just to demonstrate how to do it and also to provide a link with the price elasticity of demand.

4.4.3 Second Step: Calibration

The calibration here is different from the conventional one for interest rates because we have no compounding to deal with and hence is actually much simpler. For the first time step we have

$$p_u e^{\alpha(\Delta t)+\delta S^*} + p_m e^{\alpha(\Delta t)} + p_d e^{\alpha(\Delta t)-\delta S^*} = E[S(\Delta t)] \tag{16}$$

We have δS^* as the spacing of the tree in the price direction and the tree is for the log of the price hence the need to take the exponential. This is different from the interest rate cases although some of the formulae may seem similar it is for different reasons.

Equation (16) specifies the calibration condition at the first time step. Thus

$$\alpha(\Delta t) = \ln\left(\frac{E[S(\Delta t)]}{p_u e^{\delta S^*} + p_m + p_d e^{-\delta S^*}}\right)$$

We can generalize this directly to

$$\alpha(i\Delta t) = \ln\left(\frac{E[S(i\Delta t)]}{\sum_{j=j_min}^{j=j_max} p(i,j)e^{j\delta S^*}}\right)$$

where $p(i,j)$ is the probability of reaching node (i,j) on the tree.

The above is basically similar to the development in [Hul00] except simpler with respect to calibration. We can then use this to derive $Y(t)$ if are interested.

Note that the move from the $S^*(t)$ tree to the $X(t) = log(S(t))$ depends only on t; $\alpha(t)$ does not depend on any value of $S^*(t)$ and the probabilities on the tree are unchanged whatever the values of $\alpha(t)$. Also note that $d\alpha(t)$ is independent of $\alpha(0)$ and the original $S^*(t)$ tree is independent of $Y(t)$. These observations are at the core of the efficiency of our implementation. Each new tree we need to build after each possible option exercise requires only new values for $\alpha(t)$ and/or $Y(t)$. So we need very little extra calculation to build new dependent tress.

When an option is exercised there is an immediate drop in the price $S(t) \to S(t)/\gamma(c,C)$ where $\gamma(c,C)$ depends on the new capacity added to the market c relative to the previous capacity C. Market participants will not know C but they will be able top observe γ and c and thus work backwards to an implied C. Note that this implies that $\gamma(c,C)$ will drop as the price drops because of the capacity increase that brought about the price drop. We assume that different participants add new capacity at different times. The immediate price drop does not affect $Y(t)$. Thus we can use the original $S^*(t)$ tree with a newly calculated $\alpha'(t)$ for the (mean reverting) spot price following the price drop. In effect we can add states (or memory locations) at each node of the base tree and then keep reusing it for different spot price process calculations. This saves a lot of implementation effort and re-calculation of transition probabilities. It also has the pleasant side effect of ensuring that revenue calculations following exercise are automatically calculated for the same length of time *following exercise*. This could be achieved with other methods but this one is simple and effective.

In case it is not clear from the above, note that each new tree we build is mean reverting to the (unchanged) long term price trend from the starting point of the price drop caused by the added capacity.

4.5 Parameter Choices

For the spot price model we require $\{\eta, \sigma, \nu, \rho\}$. Note that η is the mean reversion of the log of the price not of the price itself. For the lighting cost model and the revenue model we need $\{M_0, M_{00}, K_0, l\}$, the set of lighting capacity alternatives $\{c_1, \ldots, c_n\}$, the riskless interest rate r and the difference, δ, between that and the inflation rate. We also need the parameters for supply and demand elasticity, $\{e, a(t), d(t)\}$, in order to define $\gamma(c,C)$. In fact we also use $\{e, a(t), d(t)\}$ to construct a prediction of expected mean spot price to obtain ν. Actually we use this to provide a lower limit on ν, see later.

It may appear that there are a lot of parameters. This, in fact, is typical for a real options analysis where the details of the physical investment are important and greatly increase the number of parameters relative to ordinary financial options. Many of the parameters are known or can be deduced from observations of the system. For example predictions of progress of digital technology have proved remarkably accurate over long time periods. Moore's law being probably the most extreme example.

Let us assume that the time required for prices to halve is $T_{1/2}^{price}$ and that the time required for demand — not capacity delivered in the market — is T_2^{demand}. We can now use our equations incorporating demand elasticity (Equation 1) first to obtain implied changes in supply elasticity, $d(t)$ in Equation 2, (or improvements in technology used) as:

$$
\begin{aligned}
\alpha &= \ln(2)/T_{1/2}^{price} \\
\beta &= \ln(2)/T_{2}^{demand} \\
d(t) &= \exp\left(-t\left(\beta + (e+1)\alpha\right)\right) \qquad (17)\\
q(t) &= p(t)/d(t) \qquad (18)
\end{aligned}
$$

Where we have defined α and β here (only) for convenience. e is the demand elasticity in Equation (1). Hence we can derive the increase in capacity on the market with time $q(t)$. We chose parameters so that this always doubles year on year. Likewise we arrange the economic parameters so that the expected average price halves on a two year time-scale. When we examine the role of e we have to change other parameters to keep to these two observed time-scales. Capacity changes are used in setting the price drop (γ) from adding additional capacity according to the demand elasticity, that is:

$$
\begin{aligned}
\gamma(c, q(t)) &= 1 - \frac{\text{new price}}{\text{old price}} \\
&= 1 - \left(\frac{a(t)}{q(t)}\right)^{1/e} \Big/ \left(\frac{a(t)}{q(t)+c}\right)^{1/e} \\
&= 1 - \left(\frac{q(t)}{c+q(t)}\right)^{1/e} \qquad (19)
\end{aligned}
$$

where $q(t)$ was the previous capacity on the market and c was the capacity added by lighting the dark fiber. This form (Equation 19) is also expected from the definition of a constant elasticity of demand curve.

In setting the parameters for the changes in lighting costs with time we can make use of Equation (17). Clearly technology change is not determined (in terms of cause and effect) by changes of demand and observed changes in price. These factors can easily, for example, be the results of a changing competitive situation. However in setting the decline in lighting costs it is instructive to pick values around those derived from Equation (17). The other factor in lighting costs apart from rate of decline is the base cost today.

Thus in investigating the value of dark fiber we can consider a range of base lighting costs and cost decreases relative to price declines and capacity increases. These may be either derived from simple (even simplistic) economic arguments as here or from observations or proprietary models as appropriate.

Current spot prices for bandwidth capacity are available from at least one site online (www.EnronOnline.com) and today are largely determined by the presence or absence of competition. Thus our $S0$ parameter is fairly situation dependent at present.

When considering how much capacity is added to a market by lighting fiber two things need to be taken into account. Firstly to provision many routes represented by indivisible end-to-end contracts the provider can logically shift underlying physical resources. Thus the "available capacity" for a particular market is not, even theoretically, a constant at any point in time although there is a clear upper limit. Secondly, whatever the physical capacity that is present, much will not be available because it will have been committed to long-term contracts. Thus, to see the effect on market prices from additional capacity the relevant capacity to use as the base capacity is the capacity that is competing for new contracts. This "marginal capacity" may be a very small fraction of the installed base and be shared between several different advertised end-to-end routes. Thus the best source, in practice, for γ will be observation. We use economic arguments here to give a structure to the interpretation of these observations.

Table (1) gives the parameters used. For the parameter ranges we do not attempt to display a multi-dimensional result. Instead we explore each axis relative to the same origin set of parameters. This origin set is the middle case of those considered in detail in the first results section (Section 5.1).

Many of the parameter values in practice depend specifically on which link of an overall backbone network market is considered. Additionally, price drops may have nothing to do with technology changes on many routes but simply reflect the introduction of competition. In this case the way to use the model is simply to calibrate its parameters without regard for any underlying economics. In these cases users will simply need to accept empirical usefulness rather than theoretical justification.

At EnronOnline we could observe that the (forward, for September physical delivery) price for Los Angeles to San Jose in early June 2001 was 0.0028–0.0050$/month /DS0/mile (bid–ask). This is 293 miles so repeaters etc. are not present for such a short distance (for OC192 at least). This gives roughly $100KUSD/month for OC192 service (assuming availability). Recall that OC192 = 129,000 DS0's. Note also that New York to Washington D.C. (240 miles) was quoted as 0.0007–0.0010$/month /DS0/mile (bid–ask) giving a OC192 price of $22KUSD/month, so it is clear that there are large variations in market prices at present. Both quotes were for 1 year length contracts a couple of months forward. Given that you need a mux on both ends of a fiber and these cost, say, $175KUSD for OC192 today [Cla01], this would give $K0/(S0 \times u) \approx 0.15 - 1.35$. This is taking u as 12 months. Clearly there are many caveats with this back-of-the-envelope calculation, e.g. difference between 1-year forward and spot prices, but this gives a ball-park estimate of parameter magnitude and range.

Elasticity of demand estimates for 5 year horizons are in the 1 to 1.6 range (France Telecom, Lucent Technologies and others, [LMWW00]) and [Tho00] has further estimates in the range 1 to 1.4. However these will be different on other timescales so again these number are more useful as ball-park figures for the moment in the absence of further observations.

5 Results

At this early stage of the bandwidth market given the range of different business situations the determining parameters (above) can have quite a large range of plausible values. Here we are interested in investigating how these different factors inter-relate. We will first look at a small set of parameter combinations to set the scene and then examine the effect of changing specific parameters in detail.

5.1 Base Cases

We looked at three lighting options and for these examples we chose the following parameters:

- $K0/(S0 \times u)$: 0.75; 0.75×1.2; $0.75 \times 1.2 \times 1.2$
- $k_{\text{decline}}/S_{\text{decline}}$: 1.0; 1.5; 2.0

parameter	value	interpretation
Spot Price Parameters		
$ln(2)/\eta$	0.25 – 2.00	half life of mean reversion of log(S) in years
σ	0.25 – 2.00	annualized volatility. N.B. observed volatility will be reduced by the strength of the mean reversion
ν	fit	half life of log of cost trend Y, calibrate from supply elasticity changes
ρ	0.0	uncertainty of cost trend not a focus of this work
Option Exercise Price Parameters		
$K0/S0$	0.50 – 2.00	cost per-unit-capacity today of capacity upgrades
K decline / S decline	0.50 – 3.00	time dependence of lighting cost decrease relative to rate of decline of expected spot price
M_{00}	0.05	maintenance per-unit-capacity at installation in terms of lighting cost per-unit-capacity at installation
$c0/C0$ $c0/$ previous	0.25 – 4.00 0.25 – 2.00	capacity added relative to capacity available initially ($C0$, at time zero of model) capacity added relative to capacity already owned on route by buyer. We also consider no previous capacity case.
Economic Model Parameters		
e	1.00 – 2.00	price elasticity of demand
$a(t)$	fit	demand increase with time, fix so market available capacity doubles every year
$d(t)$	fit	price elasticity of supply (new technologies decrease d), fix so expected observed prices halve every two years
r	0.05	riskless rate of interest, 5%

Table 1: Parameters for the spot price model, option exercise price models (exercise and maintenance), and for the basic economics of supply and demand.

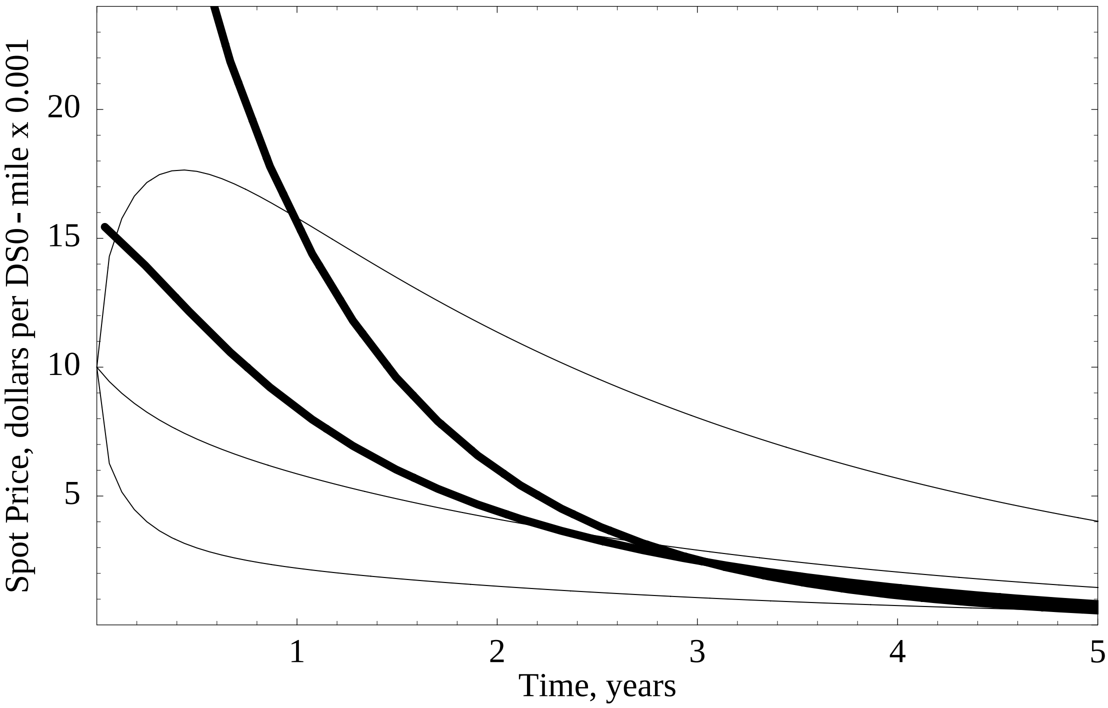

Figure 5: Optimal exercise boundaries for different lighting capacity options (thicker lines; upper 4× added capacity of lower and upper is 20% more expensive per-unit-capacity initially, but relative cost decreases 33% faster). Expected option exercise times are 0.94 ± 0.85 years and 1.90 ± 0.80 years respectively for lower and higher capacity options. Light lines are confidence intervals for spot price process. Percentiles shown are 5%, 50% (median) and 95%.

- c/C: 0.25; 1.0; 4.0 relative to capacity available at start (t=0). Recall that just as costs are decreasing with time capacity on is increasing with time.
- e: 1.25.

The progression in exercise cost with capacity increase is in accordance with industry comments that currently OC768 is somewhat proportionately more expensive today than OC192 but that this cost expected to drop. We considered lighting options being, 0.25 units, 1 unit and 4 units respectively. We considered these additions with respect to a previously available capacity at the start of 1 unit. Note that capacity on the market doubles each year given the demand parameters we have chosen. The volatility was 100%, this is consistent with other markets for non-storeable commodities (electricity where there is no hydro or pumped storage, for example). Results were as follows for this example

- Option Value (in units of $S0 \times u$, where u is one year): $1.24 \times 0.25 = 0.31$; $0.87 \times 1.0 = 0.87$; $0.65 \times 4 = 2.60$
- Mean First Passage Time (FPT, or expected option exercise date): now; 0.94 ± 0.85 years and 1.90 ± 0.80

Figure 5 shows the optimal exercise boundaries for the two larger lighting capacity options relative to spot prices. The time when the observed spot price intersects with each boundary is the optimal point at which to exercise the respective lighting/upgrade option. The time is not fixed because the spot price is stochastic. We can calculate a mean first passage (exercise) time for each option and its standard deviation (although this should be treated with caution because the first passage — option exercise — time distribution is not symmetric).

The smallest lighting choice should be exercised immediately to obtain maximum return for that lighting option. More expensive and higher capacity lighting options become optimal to exercise as their relative price and capacity decreases and depend on the spot price development. The advantage of upgradeability for dark fiber is that some capacity can be lit early and later capacity added as market conditions change. This has a large effect on both cashflow and value. Cashflow starts earlier but larger lighting capacity options can still be exercised to capture the value of increased demand — even at much lower prices.

Now if we take the case where the buyer of the dark fiber already has lit fiber on the same route we need to factor in the loss of revenue caused by the price drop caused by lighting the new capacity. This changes the previous results to the following for a previous unit capacity position.

- Option Value (in units of $S0 \times u$, where u is one year): $0.91 \times 0.25 = 0.22$; $0.72 \times 1.0 = 0.72$; $0.63 \times 4 = 2.52$

- Mean First Passage Time (FPT, or expected option exercise date): 0.25 ± 0.46; 1.66 ± 0.84 years and 2.18 ± 0.46

The value drops least for the highest relative capacity addition, as expected. However in all cases the mean exercise date is delayed, and in the case of unit addition (middle case) nearly doubled from 0.9 years to 1.6 years. Thus, whilst the later cashflow may be little affected by a sequence of dark fiber lighting and capacity upgrades, the early and intermediate cashflows take a significant impact. Clearly the player able to pay the most for dark fiber will always be the one with no prior position all other factors being equal.

We now look at individual parameter effects in detail using as our "origin case" the middle example given here.

5.2 Exercise Price, Volatility and Mean Reversion

Figure 6 shows dependence of option value (left panels) and mean option exercise date (in years from start, right panels) on: option exercise price; spot price volatility; and half-life of mean reversion of log spot price. Note that the $K0/S0$ ratio is the ratio at $t = 0$. The spot price of bandwidth changes with time and so do exercise costs.

The high dependence of option value on exercise price (i.e. $K0/S0$ ratio) is no surprise (Figure 6, top-left) nor are the corresponding changes in expected option exercise date (Figure 6, top-right). At a low enough exercise price ($\approx 0.7 K0/S0$) the optimal strategy is to exercise at once. This may seem a little strange until the nature of a lighting action is recalled: after lighting the owner receives a continuous stream of revenue (until maintenance costs make the equipment uneconomic). Thus, providing the initial cost is low enough, the earlier revenue outweighs the penalty of not having the exercise cost even lower as lighting equipment costs decrease. For high equipment costs (at $t = 0$) waiting is necessitated but even then the eventual benefit is low (Figure 6, top-right).

Spot price volatility plays a smaller role than exercise price (Figure 6, middle). This is partly due to the mean reverting nature of the spot price. Whatever the volatility parameter the spot price will have its volatility limited by the reversion of the spot price to the mean. The other limitation on the significance of volatility is that this is relative to the mean price which is decreasing. However, even if the option value is little affected the mean exercise date does have a strong dependence. As usual this is because increasing volatility improves the chances of higher prices later.

Mean reversion 6, bottom) mirrors the effects of volatility on exercise date but has an opposite effect on option value. Basically a longer reversion time makes it better to wait because of the chance of higher prices. However, unlike the effect of volatility, the penalty of

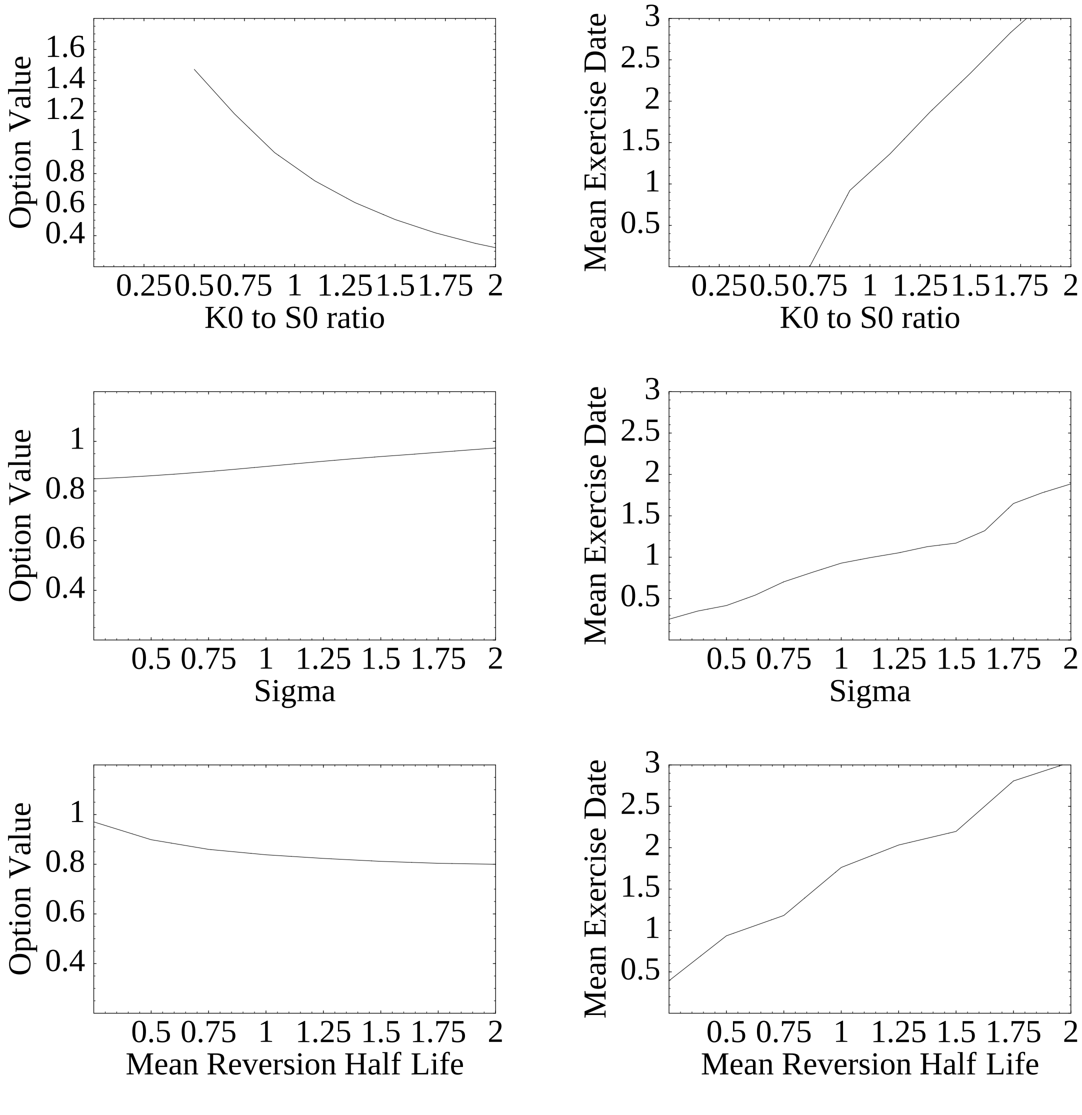

Figure 6: Option value per unit of added capacity (left panels) and mean option exercise date (in years from start, right panels) dependence on: option exercise price (*top*); spot price volatility (*middle*); and half-life of mean reversion of log spot price (*bottom*).

a lower mean price outweighs this benefit. It is necessary to wait longer to maximize option value but this does not compensate for the price drop penalty incurred. Essentially it takes too long for the benefits on the price from longer reversion times to take effect.

5.3 Elasticity and Price / Spot Decrease Ratio

Figure 7 shows dependence of option value (left panels) and mean option exercise date (in years from start, right panels) on on demand elasticity and on the ratio of rate of decrease of option exercise price to rate of decrease of mean spot price.

When demand elasticity is altered, as here (Figure 7, top), changes in $d(t)$ are required so that the mean price is unchanged. We do this so that the results of all the different sections can be compared on the basis of the same mean price development. Given this, increasing demand elasticity has only a slight benefit to option value and mean exercise date.

In contrast to demand elasticity, the ratio of the rates of decrease of lighting costs (K decline) to mean price drop (S decline) have a much larger effect on the mean exercise data (Figure 7, bottom) although only a small effect on option value. Below a certain K decline to S decline ratio (≈ 1.1) the optimal strategy is to exercise at once. This is simply because the option exercise price will not get more attractive later. On the other hand the inverse is also true: above a certain value there is little benefit to further delay in lighting. This is because after a certain point the cost of lighting is so low it no longer matters much what the price

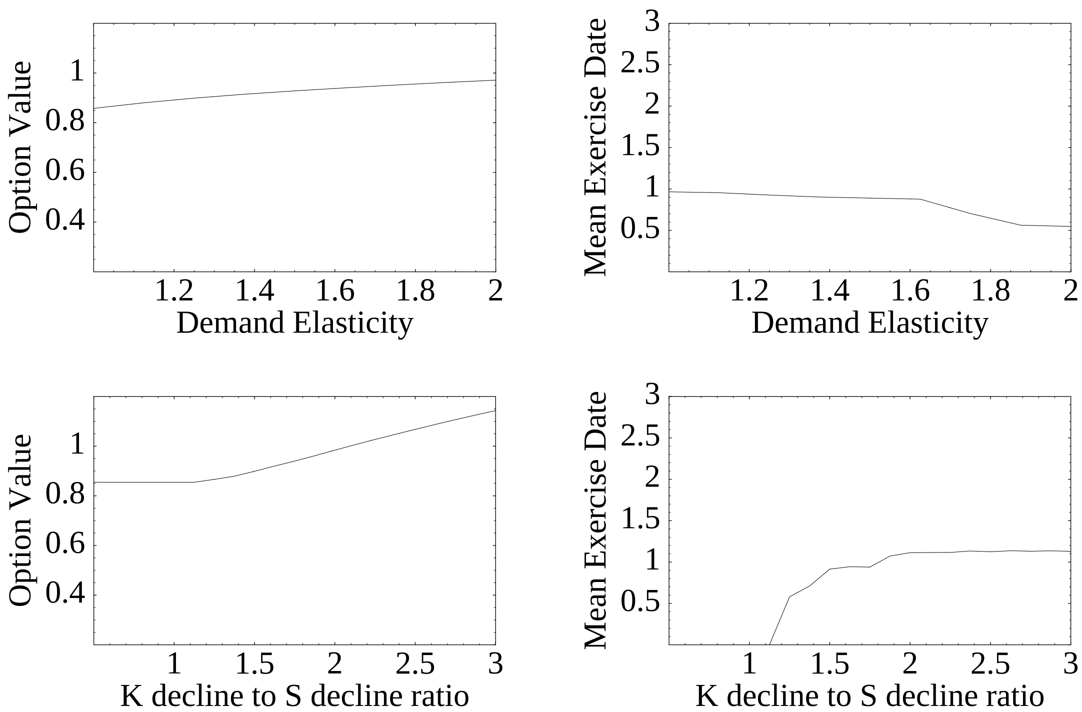

Figure 7: Option value (left panels) and mean option exercise date (in years from start, right panels) dependence on demand elasticity (*top*) and on the ratio of rate of decrease of option exercise price to rate of decrease of mean spot price (*bottom*).

is doing. Clearly this is somewhat unrealistic. In reality above some ratio there would be a knock on effect on prices. We have modeled this in our earlier section on basic economics but have chosen to treat factors separately here for clarity.

5.4 Capacity Added and Previous Capacity Owned

Figure 8 shows dependence of option value (left panels) and mean option exercise date (in years from start, right panels) on relative capacity added by option exercise. Note that this ratio is taken at the start. Dependence on the ratio capacity added to previous capacity already owned is below.

If option exercise adds greatly to bandwidth market size on a given route then the benefit will be reduced on a per-unit-capacity basis and actual exercise will be pushed back (Figure 8 top). However, it is also obvious that since much more capacity is delivered, the option value on a per-option basis is much higher as this is per-unit-capacity×capacity.

If the buyer of dark fiber already owns lit fiber on the same route the value of lighting will be reduced (Figure 8 bottom). Adding capacity reduces prices and so an existing revenue stream will be reduced. This will affect option exercise value and it changes the expected option exercise date. Given a smaller and smaller previous capacity position (increasing c0 to Previous Ratio) there will be less and less effect on option value.

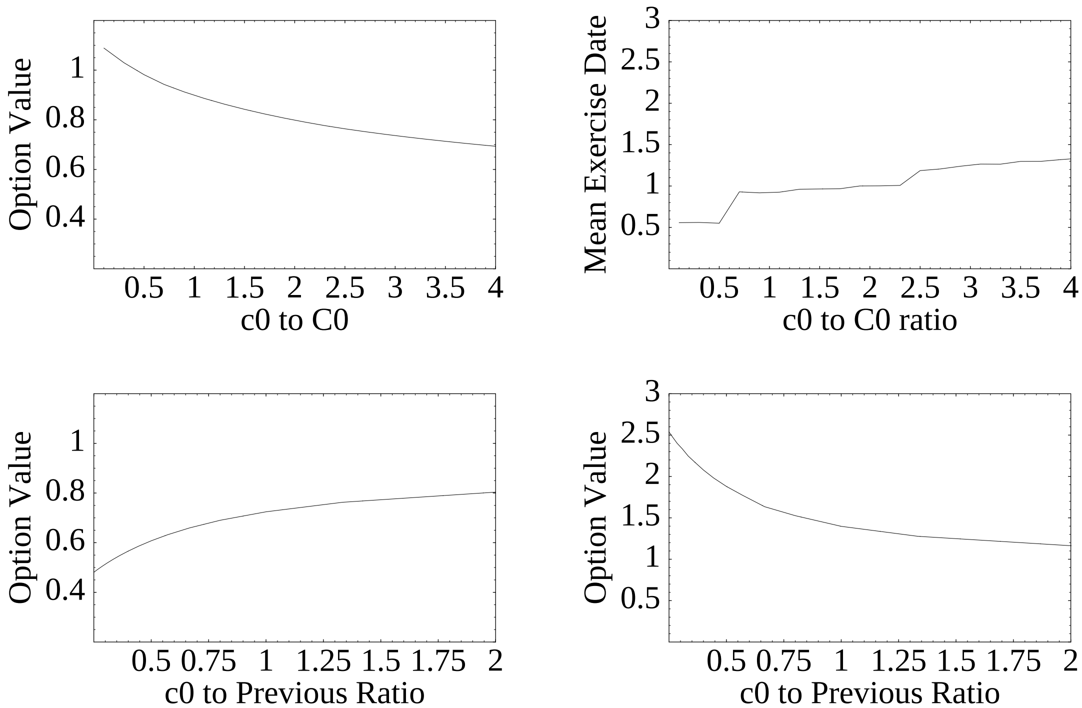

Figure 8: Option value (left panels) and mean option exercise date (in years from start, right panels) dependence on relative capacity added by option exercise (*top*). Note that this ratio is taken at the start. Dependence on the ratio capacity added to previous capacity already owned is below (*bottom*).

6 Discussion

We have presented here an application of real options in bandwidth markets: the valuation of dark fiber. Dark fiber may be lit at any time during the owners' indefeasible right of use, this is the first option. This lighting may also be upgraded at any time giving further options. What are these options worth? This depends on the revenue that they make possible from sales of capacity on spot or forward markets. These growing bandwidth markets start to make possible valuation of dark fiber as options on this (stochastic) revenue stream simply because market prices are now becoming available. Thus we have the two ingredients for a real options analysis: an asset with decision possibilities (irrevocable investments), and a market (transparent prices). This is what we have demonstrated here, the valuation of dark fiber using details of the asset (e.g. decreasing technology cost for increasing lit capacity) and a stochastic process for the market prices (adapted to bandwidth, i.e. decreasing and mean reverting).

We have shown the dependence of dark fiber value on a set of key parameters: lighting cost; rate of lighting cost decrease; price volatility; strength of mean reversion of prices. We have explicitly included: the rate of growth of capacity in the market; demand elasticity; and the effect on market prices of adding new capacity to the market. Finally we looked at how the dark fiber value is changed for market participants who already have capacity in the same market. As well as investigating dark fiber value we also looked at when the lighting option would be exercised, the expected option exercise time (or mean first passage time). This is relevant because even if lighting later has a high value there may be interest in having an earlier start to the revenue stream. Alternatively it is useful for capital budgeting to have an awareness of when capital investments may be required to optimally exploit assets. Option value is reduced almost in direct proportion to increases in lighting cost and option exercise is similarly delayed. The dependence of option value on spot price volatility or reversion strength is much less but the delay in option exercise is similar to that from increased lighting costs. In terms of rate of decline of lighting costs: below a certain threshold there is no incentive to wait, the option should be exercised immediately. Given that market prices are decreased by additions of capacity it is no surprise to observe that dark fiber is less valuable the more previous capacity is owned: its price gets reduced too. Also the more significant the addition to the market (assuming a competitive market) the less that capacity is worth (on a per-unit-capacity basis). These last results clearly match real market observations, they confirm that this valuation model includes significant (non-linear) features not present in typical financial market option models.

The valuation model presented here is limited in that it only considers the first lighting decision. A more complex model would also included the compound nature of lighting upgrades. This is a relatively simple addition to the current model. However, because market prices are changed by addition of capacity, the valuation tree is semi-bushy and this would require much longer computation times. We did include chooser options, i.e. you may chose which of a set of lighting choices to make as the first lighting decision. Under most parameter combination this had no effect on the results. The reason for this is that capacity options come in 1:4:16 ratios, thus adding a larger capacity multiplies the profit by a similar factor when it becomes optimal to light. We found that this strategy dominated. In a compound option situation what we would expect is that there would be an early exercise of a low capacity option with the same later large capacity addition. The effect can be roughly approximated by taking pairs of single options and simply increasing the price of the second by the revenue lost from the first after the later upgrade. We found that this did produce earlier revenue and a small increase in dark fiber value.

We valued dark fiber taking into account only the price process for a single route: the route where the dark fiber was. In fact the situation is a little more complex. The network nature of the commodity means that changing prices on one route will affect prices on other routes and this in turn will have an effect on prices on the original route. These effects have been studied in detail in [KC01b, KC01a, CK01] and are used (without the feedback) in [Kep01]. Including these effects in dark fiber valuation would require a different method because there is no closed form expression available which includes these feedback effects. The way to do it in this case would be to use a combined simulation and dynamic programming option valuation method such as [LS01, TR01, BG98]. This level of detail was not the objective here. Note that even if the buyer has no capacity on the route that the dark fiber follows, by changing the prices on that route the prices of other routes will be affected and thus investment value on those other routes. Again this was beyond the scope of the current work.

In summary, we have presented a model for dark fiber valuation taking into account both technological, economic, and market factors. Particularly important details unique to this analysis are that prices are stochastic and decreasing and that addition of new capacity in a market situation changes prices according to demand elasticity. Further applications of real options in bandwidth markets will appear as the bandwidth market, and associated online markets, develop.

A Appendix: Trinomial Tree Details

These are as in [Hul00]. For the tree we take $\Delta S = \sigma\sqrt{3\Delta t}$. Let each node of the tree be (i, j) and $j_{\max}, j_{\min}$ be where we change branching pattern. We chose $j_{\max} = -j_{\min} = \lceil 0.184/(\eta\Delta t)\rceil$. The branching probabilities depend on which type of branching is going on as follows:

	p_u	p_m	p_d
usual	$\frac{1}{6} + \frac{\eta^2 j^2 \Delta t^2 - \eta j \Delta t}{2}$	$\frac{2}{3} - \eta^2 j^2 \Delta t^2$	$\frac{1}{6} + \frac{\eta^2 j^2 \Delta t^2 + \eta j \Delta t}{2}$
up	$\frac{1}{6} + \frac{\eta^2 j^2 \Delta t^2 + \eta j \Delta t}{2}$	$(-\frac{1}{3} - \eta^2 j^2 \Delta t^2) - 2\eta j \Delta t$	$\frac{7}{6} + \frac{\eta^2 j^2 \Delta t^2 + 3\eta j \Delta t}{2}$
down	$\frac{7}{6} + \frac{\eta^2 j^2 \Delta t^2 - 3\eta j \Delta t}{2}$	$(-\frac{1}{3} - \eta^2 j^2 \Delta t^2) + 2\eta j \Delta t$	$\frac{1}{6} + \frac{\eta^2 j^2 \Delta t^2 + \eta j \Delta t}{2}$

Where p_u is the probability of the highest branch, p_m that of the middle branch, and p_d for the lowest branch.

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