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Research Report

Application of Capacity-Approaching Coding Techniques to Digital Subscriber Lines

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Abstract- **The use of coding for error control is an integral part in the design of modern communication systems. Capacity-approaching codes such as turbo and low-density parity-check (LDPC) codes, discovered or rediscovered in the past decade, offer near Shannon-limit performance with rather low implementation complexity and are therefore increasingly being applied for error control in various fields of data communications. This paper describes a generic multilevel modulation and coding scheme based on the use of turbo, turbo-like, or LDPC codes for digital subscriber line (DSL) systems. Such techniques are also suitable for general multilevel modulation systems in other application areas.**

I. INTRODUCTION

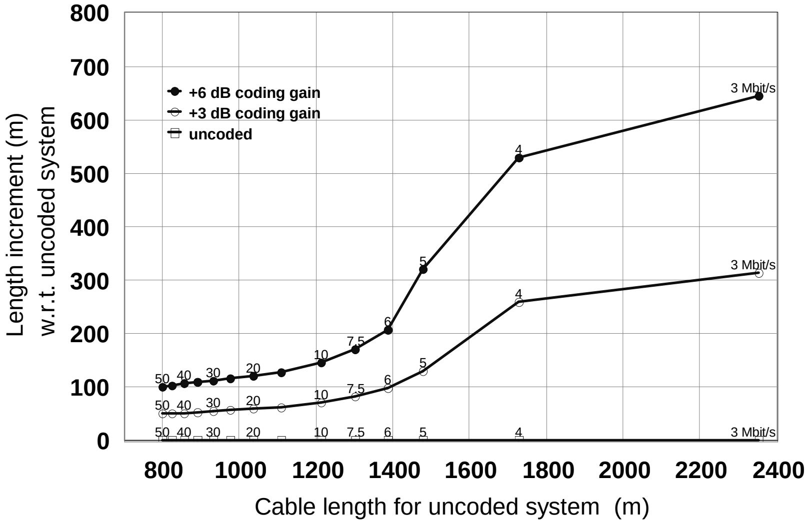
Error-correcting codes have played an important role in achieving high data integrity in transmission systems. The T1.413 asymmetric digital subscriber line (ADSL) specification published by the American National Standards Institute (ANSI) in 1995 was the first DSL standard to incorporate error-correction coding. This ANSI document specifies the use of Reed–Solomon (RS) coding with code symbols from GF(2^8), together with symbol-level convolutional interleaving, as a forward error-correction technique for ADSL systems that employ discrete multitone (DMT) modulation. Coding redundancy, number of DMT frames per RS codeword, and depth of interleaving are the parameters selected from a pre-defined set of values to provide the best possible match to the transmission-channel characteristics and the application-specific constraints [1]. The ANSI document also includes the optional use of a 16-state, four-dimensional trellis-coded-modulation (TCM) scheme as an inner coding mechanism to improve communication reliability further. Subsequent ADSL specifications have retained this coding structure, with some variations on the set of allowed parameter values. The current Very-high-speed DSL (VDSL) standards [Standards Committee T1 - Telecommunications: T1.424, European Telecommunications Standards Institute (ETSI): TS 101 270] has only included outer RS coding, whereas the Single-pair High-speed DSL (SHDSL) specification [International Telecommunication Union - Telecommunication Standardization Sector (ITU-T): G.991.2] has only included inner trellis coding without an outer error-correction code. Note that the latest G.992.3 ADSL2 Recommendation by ITU-T makes the use of inner TCM mandatory for upstream as well as downstream transmission.

Today, as DSL deployments proceed at a rapid pace, local exchange carriers are gaining field experience and therefore are interested in optimizing their plant usage and in operating their equipment closer to performance limits. The incorporation of powerful near Shannon-limit coding techniques, such as the ones discussed in this paper, offers a path toward this goal by allowing higher service penetration (loop reach), higher data rates, and more robust system operations. In particular, the use of turbo and LDPC coding is already under consideration for DSL technologies. Various coding schemes for multilevel transmission have been proposed to ITU-T and to Committee T1 with the aim of enhancing the performance of the existing ADSL standards as well as of the evolving VDSL standards.

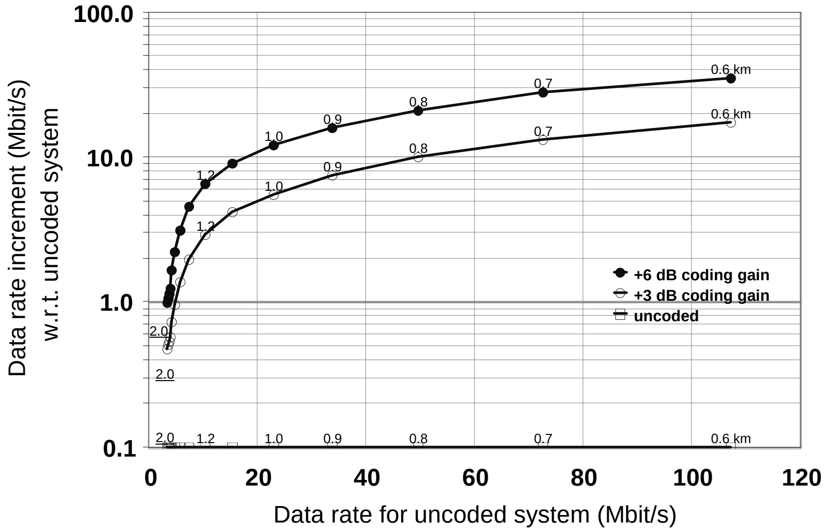
In this paper, the various approaches in applying capacity-approaching coding techniques to DSL systems are presented. More specifically, turbo coding [2] and low-density parity-check (LDPC) coding [3] are considered, and the performance improvements that can be expected from the introduction of such techniques into DSL systems, in terms of both loop reach and data rate, are discussed. A generic scheme for implementing turbo- and LDPC-coded modulation is then described, and examples of the performance achieved by the approaches proposed are given. Comparisons with the current ADSL TCM standard are also provided, both in terms of performance and implementation complexity. It is demonstrated that the introduction of advanced coding and iterative decoding techniques offers the promise of operating DSL links much closer to capacity without incurring a substantial increase in transceiver complexity.

II. CODING FOR DIGITAL SUBSCRIBER LINES

The following example illustrates the performance benefits that can be expected from applying coding techniques to DSL systems. Consider a “category-3” voice-grade unshielded twisted-pair cable with a transfer function having a $\sqrt{f}$ dependency on frequency. Signal attenuation is assumed to be equal to 75 dB at 5 MHz for a cable length of 1 km. Signal disturbance is caused by additive white Gaussian noise (AWGN) with a power spectral density of –140 dBm/Hz and alien near-end crosstalk (NEXT) due to 49 ADSL (ITU-T, Recommendation G.992.1) downstream transmitters that use all the subcarriers. By performing channel capacity computations, the data rates that are achievable on this channel are determined, assuming a transmit power of 15 dBm, a signal-to-noise-ratio (SNR) margin of 6 dB and a gap to capacity for uncoded transmission of 9.95 dB, which implies operation at a symbol error rate of 10^{-7}. The results of these computations are shown in Fig. 1 in terms of the increase in data rate and loop reach obtained with a coding gain of 3 and 6 dB with respect to an uncoded system. For example, Fig. 1a shows that, for operation at 3 Mbit/s, the loop reach of an uncoded system is limited to 2355 m. If coding is employed with a net coding gain of 3 dB (6 dB), the loop reach can be extended by 315 m (645 m). As another example, Fig. 1b shows that, for a loop reach of 2000 m, an uncoded system can achieve a data rate of 3.5 Mbit/s and that a coding gain of 3 dB (6 dB) allows this data rate to be increased by 480 kbit/s (1 Mbit/s). Similar results can be derived for a variety of channel and noise models.



(a)



(b)

Fig. 1. Example of performance improvement due to coding for DSL transmission. (a) Cable-length increase achieved by coding. The numbers above the curves indicate achievable data rates in Mbit/s. (b) Data-rate increase achieved by coding. The numbers above the curves indicate cable length in km (the data-rate increment for the reference uncoded system is artificially displayed at the ordinate value of 0.1 instead of zero because of the logarithmic scale).

It is readily seen that coding can play an important role for increasing the performance of DSL systems in terms of data rate and loop reach. As will be shown in this paper, coding gains of 6 dB and more can be practically achieved through the use of capacity-approaching coding techniques. As an example, the performance of a particular LDPC code mapping 2021 information bits to 2209 encoded bits is illustrated in Fig. 2. The spectral-efficiency versus power-efficiency of this code is shown for three different quadrature-amplitude-modulation (QAM) constellations at a bit-error rate (BER) of 10^{-7} (triangles). Power efficiency is measured by E_b/N_0, the ratio of energy per-bit to noise power-spectral density. Fig. 2 also includes the capacity of the QAM signal sets (squares are indicated on these curves at the same values of spectral efficiency as for the LDPC-coded cases), shedding light onto the effectiveness of the multilevel LDPC coding scheme. Note that the gap in E_b/N_0 between the capacity limits and the power efficiency of the LDPC schemes remains fairly constant, and is nearly independent of the spectral efficiency. Finally, the circles and stars in Fig. 2 indicate the values of E_b/N_0 for which a $\mathrm{BER} = 10^{-7}$ is achieved for TCM and uncoded M-QAM, respectively.

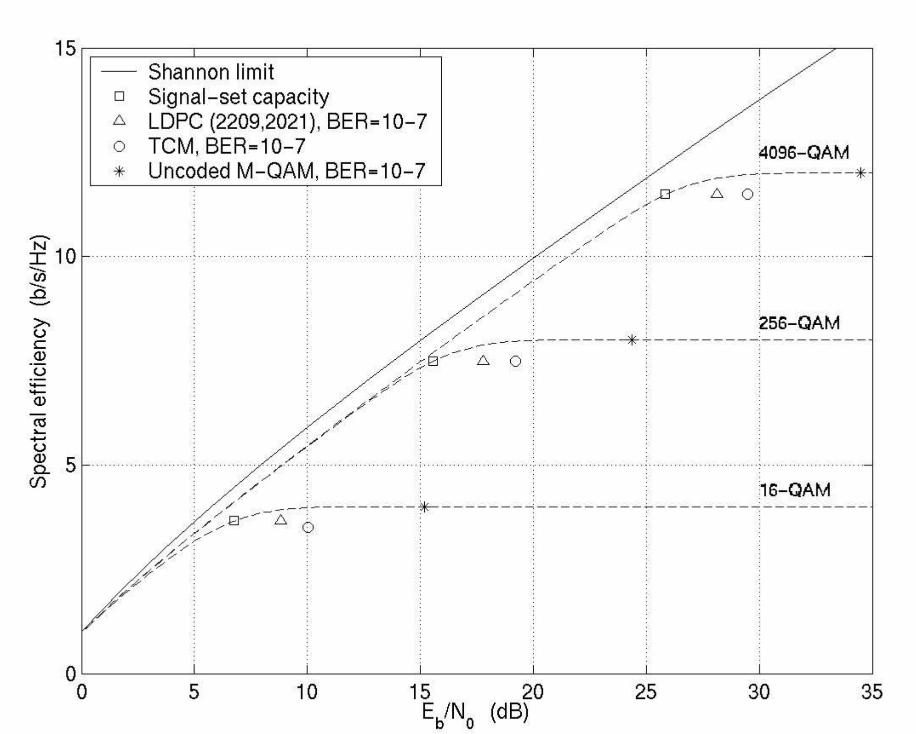


Fig. 2. Example of LDPC code performance at different spectral efficiencies.

Code design should in general take into account the specific constraints introduced by the application. In particular for DSL systems, there are three important aspects that must be considered in designing the inner and outer coding schemes. First, high code rates are obviously desirable to achieve high spectral efficiencies for bandwidth-constrained DSL transmission. Second, it should be possible to adapt the code parameters to given transmission-channel characteristics and application-specific constraints to achieve best performance. A simple solution would consist in allowing the receiver to select the most appropriate code from a small set of pre-defined codes following the channel-measurement phase during transceiver initialization. An approach that appears to be even more attractive, because of its greater flexibility, is the one that would allow code construction "on the fly," provided that the processing effort needed to compute interleaver patterns for turbo codes, or parity-check matrices for LDPC codes, is small. In such a case the receiver should ideally need to convey only a small number of parameters to the transmitter for code specification. A third aspect is the linear-time encodability. This very important property, which is natural in turbo coding, should also be achieved by LDPC codes.

In DSL transmission, overall delay, or latency, is also a critical issue. “Voice” applications are known to demand rather low latency whereas other applications, such as video streaming, tolerate larger delays but need stronger error-correction capability. Thus, in studying new coding techniques for DSLs, trade-offs between coding gain and latency have to be well characterized. Furthermore, the coding gains achievable also depend on the number of iterations that are allowed for soft decoding, which is tied to the $250\,\mu$s frame period adopted for multicarrier-modulation-based DSL systems [1].

Finally, another important issue is the implementation complexity associated with coding. Complexity is a critical parameter, especially at the central-office access multiplexors or at remote terminals, because it directly affects equipment cost and power consumption.

In the following sections, transmitter and receiver functions for capacity-achieving coding in DSL systems are described. The family of coding schemes discussed includes the traditional parallel concatenated turbo coding, serial concatenated turbo coding, as well as LDPC coding. The important question of whether these coding schemes should be part of a concatenated structure with outer RS coding is a topic of intensive current research, especially in terms of the impulse-noise resilience properties of the coding schemes involved.

III. Description of Transmitter and Receiver Functions

The block diagram of Fig. 3 shows the structure of a generic transmitter and receiver that will serve as a basis for our discussion. Information bits representing data or control messages are encoded into a binary codeword of length *N*. Thus, both the turbo- and LDPC-coding schemes are regarded as rate-*K/N* binary block codes. For turbo coding, the number of information bits *K* corresponds to the size of the interleaver. The symbol mapper collects groups of coded bits, possibly along with uncoded information bits, and builds complex frequency-domain-modulation symbols in compliance with the results of a bit-loading algorithm. Frames of QAM symbols obtained in this way are processed by an inverse discrete Fourier transform (IDFT) operation to yield real time-domain transmit signals.

The number of uncoded and coded bits per transmit symbol is a design parameter that involves a trade-off in terms of performance and decoding complexity. This parameter can, for example, be specified by the receiver for each symbol set during initialization. Note that it is convenient to consider symbol sets that correspond to square-shaped constellations. In that case, the real and imaginary parts of the received noisy complex signals can be demapped independently, which leads to a significantly lower complexity. In particular, for a 2^{2b}-point signal set, b = 1, 2, …, the number of decision regions is reduced from 2^{2b} to 2^{b+1}. Thus, for transmitting a total of $b = b_u + b_c$ bits per dimension, where b_u and b_c denote the number of uncoded and coded bits, respectively, mapping to complex transmit QAM symbols is used, where each complex symbol is obtained by independent L-ary modulation, $L = 2^b$, along the real and imaginary axes.

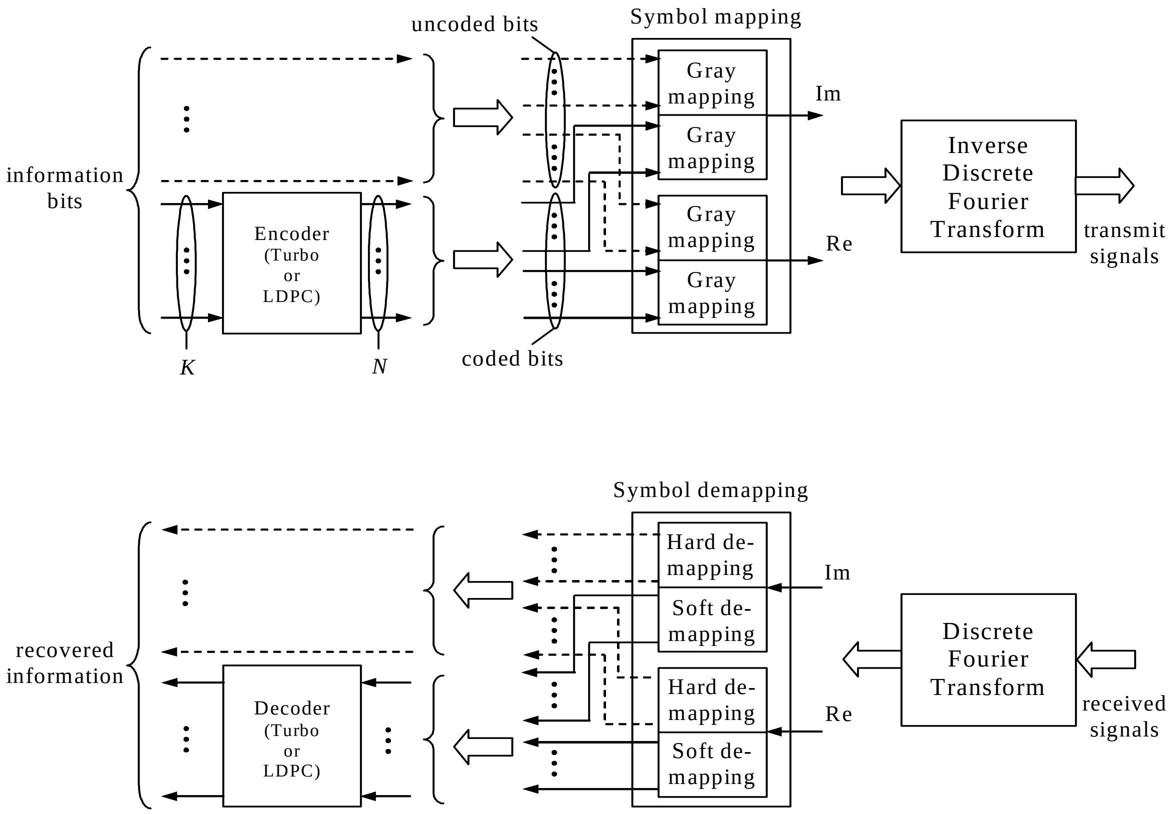


Fig. 3. Encoding/symbol mapping and symbol demapping/decoding functions for DMT-based DSL transmission.

For example, in ADSL systems, the IDFT modulator generates DMT frames at the rate of 4000 Hz. Assuming that 1024-QAM is employed on each subcarrier, with a total number of 200 subcarriers, and that $b_c = 3$ coded bits are carried per dimension, a codeword of length N = 6 bits × 200 = 1200 bits along with 4 bits × 200 = 800 uncoded bits can be mapped into a DMT frame. The line data rate in that case is 10 bits × 200× 4000 Hz = 8 Mbit/s.

In the scheme of Fig. 3, symbol mapping relies on the partition of the set of L-ary symbols into 2^{b_c} subsets such that the minimum Euclidean distance between the symbols within each subset is maximized. Gray-code labeling is adopted for the b_c less significant bits on which the soft demapper needs to generate reliability information. Furthermore, as the more significant bits are obtained via simple threshold detection at the receiver, labeling those bits with a separate Gray code within each subset permits to lower the BER. We note that Gray-code labeling is optimum in an information-theoretic sense as it leads to the largest capacity for bit-interleaved coded modulation [4]. As will be demonstrated in the next section, the double Gray-code labeling adopted here represents a good trade-off in terms of achievable performance and implementation complexity.

At the receiver, the complex noisy symbols obtained at the output of the discrete Fourier transform are processed by a demapper, whose function is to generate soft reliability information on the individual code bits and hard decisions on the uncoded information bits. Reliability information can take the form of *a posteriori* probabilities (APPs) or (log-) likelihood ratios. It is used for soft iterative decoding by the BCJR algorithm [2] in the turbo-coding case or the sum-product algorithm [3], [6] in the LDPC-coding case, or by some reduced-complexity decoding scheme. The recovered information bits are finally output from the receiver.

The above description suggests the possibility of defining a generic coding scheme for DSL systems that incorporates both turbo- and LDPC-coding capabilities. In this approach, a transmitter would be able to perform the encoding functions for both types of codes efficiently, whereas a receiver would indicate the coding technique to be employed during initialization. The clear advantage of our approach would be the increased freedom for receiver design.

IV. PERFORMANCE

In this section, simulations results are presented that illustrate the performance that can typically be achieved by the turbo-coding and LDPC-coding schemes described above. As mentioned earlier, the telephone-twisted-pair channel introduces frequency-dependent signal distortion as well as several other forms of disturbances, of which crosstalk is the most important. In the following, only AWGN disturbance will be assumed. The reason for this is that if each DMT subchannel has a sufficiently narrow bandwidth, then each one independently approximates an AWGN channel with a particular SNR value. Note that impulse noise and narrowband interference of various origins, e.g., AM radio signals, also affect the reliability of communications in DSLs. Performance should ultimately be assessed using actual test-loop conditions, but, for space reasons, will be limited here to the AWGN-channel case.

To evaluate performance, both uncoded and coded systems are represented in terms of symbol-error rate (SER) versus the normalized signal-to-noise ratio $\mathrm{SNR}_{\mathrm{norm}}$, which, for a modulation and coding scheme operating at given rate η (in bits per two-dimensional symbol), is defined as [6]

$$\mathrm{SNR}_{\mathrm{norm}} = \frac{\mathrm{SNR}}{2^{\eta}-1} = \frac{\eta}{2^{\eta}-1}\frac{E_{\mathrm{b}}}{N_0} .$$

Note that, in the case of uncoded M-QAM transmission, $\eta = \log_2 M$ and the SER can be expressed as

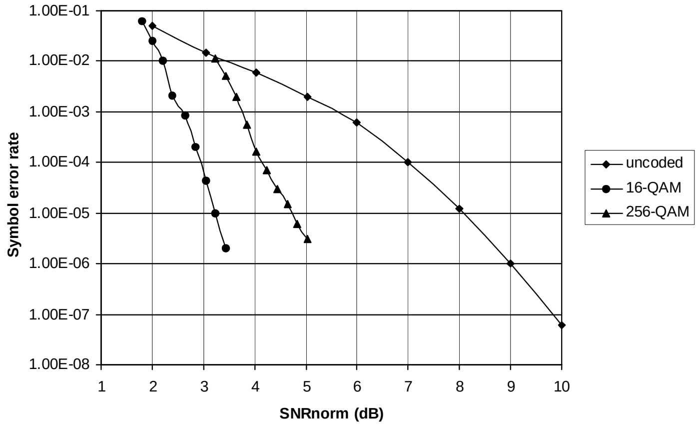
$$P_S(E) \approx 4\,Q(\sqrt{3\,\mathrm{SNR}_{\mathrm{norm}}}\,) , \qquad \text{with} \quad Q(x) = (1/\sqrt{2\pi}\,)\int_x^{\infty} \exp(-z^2/2)\,dz ,$$

which is nearly independent of the constellation size provided the latter is sufficiently large. Note also that for a capacity-achieving scheme that transmits C bits/symbol, $\eta = C$ and $\mathrm{SNR} = 2^C - 1$, which implies that $\mathrm{SNR}_{\mathrm{norm}} = 1$ (0 dB). Hence the curve of $P_S(E)$ versus $\mathrm{SNR}_{\mathrm{norm}}$ also indicates the "gap to capacity" at a given SER. For example, for an uncoded M-QAM system operating at the SER of 10^{-7}, the gap to capacity is approximately 9.95 dB.

TURBO CODING

Various coding and decoding approaches according to the turbo-coding principle introduced in [2] have been proposed in the literature. In particular, parallel or serially concatenated convolutional codes, repeat accumulate codes and other combinations thereof have been shown to approach the capacity of the AWGN channel. The common features of all these approaches are the interleaving and deinterleaving functions as well as the soft-input soft-output APP decoder usually implemented by the BCJR algorithm [7], [8].

In this paper, turbo coding with two component codes similar to the architecture introduced in [2] was used. Each constituent code is generated by an 8-state recursive systematic convolutional (RSC) encoder with feedback and feedforward polynomials equal to $(15)_{\mathrm{oct}}$ and $(17)_{\mathrm{oct}}$, respectively. For each M-QAM constellation $(M \geq 16)$, one parity bit is used in each dimension from one of the RSC encoders, and the remaining parity bits are punctured. No uncoded bits are used for signal mapping. Hence, the code rates for 16, 256, and 4096 QAM are 0.5, 0.75, and 0.833, respectively. Fig. 4 shows the SER performance of the turbo-coding scheme with three different interleaver



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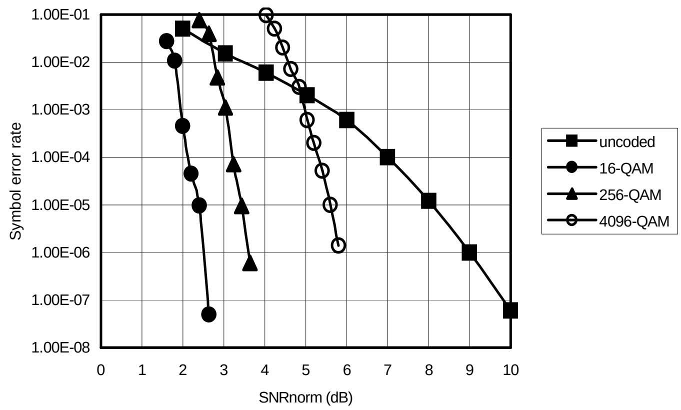

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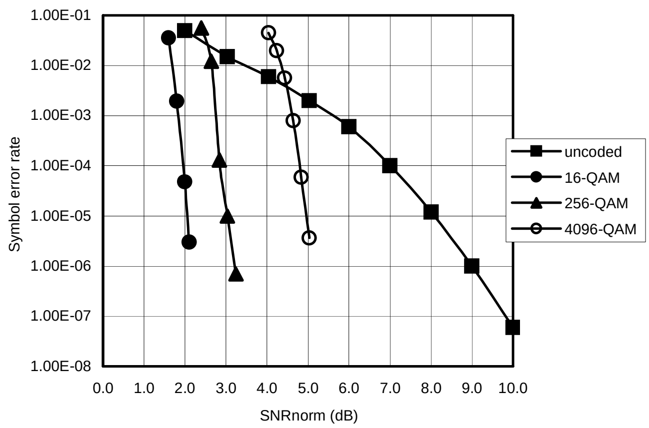


(c)

Fig. 4. Performance of three turbo codes with an interleaver length of (a) 462, (b) 2022, and (c) 4224 bits for transmission over the AWGN channel using 16, 256, and 4096-QAM.

lengths of K = 462, 2022, and 4224 bits. A semi-random interleaver is used in all theses cases. For decoding, a total of 20 iterations of the log-MAP algorithm is performed. Because of the high puncturing rate (nine out of every 10 parity bits are punctured from each encoder), the performance in the case of the 4096-QAM constellation with an interleaver length of 462 is worse than that of the uncoded system. The performance can be improved substantially by using uncoded information bits in the construction of large-size symbol constellations, as indicated in the preceding section. This approach will reduce the puncturing rate and require less computational complexity for decoding. For example, a coding gain of 7.1 dB is reported in [9] for a 16384-QAM with code rate of 12/14 and interleaver size of 2000 bits for only eight iterations. Another observation is the fact that the SER performance of turbo codes at different constellation sizes differs because of the different coding rates and puncturing patterns for these constellations. Clearly, the performance of the turbo codes can be improved by increasing the number of states for each component code.

As mentioned above, DMT frames in ADSL systems are generated at the rate of 4000 Hz. Therefore if K information bits are encoded into one DMT frame, the encoding and decoding functions introduce a latency of 250 μs each, resulting in a total latency of 0.5 ms. If a codeword spans more than a single frame, latency is increased accordingly.

To determine the net coding gain as a function of coding latency, we consider a DMT system with a total number of 100 or 200 subcarriers and two constellation sizes of 16 and 4096-QAM. The results summarized in Table 1 show the net coding gains of turbo codes in dB at a SER of 10^{-7} for different values of coding latency (no outer RS code). For these simulations the maximum interleaver size was 4800 and 12000 bits for 16 and 4096-QAM, respectively.

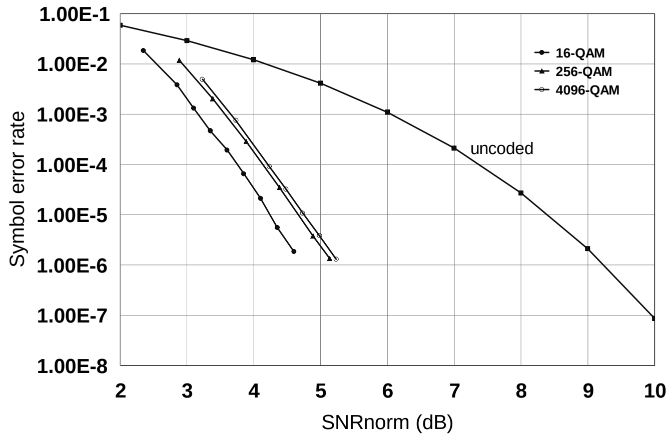
Table1. Net coding gains (in dB) achieved by selected turbo codes as a function of latency

Modulation	**# of sub-carriers**	**Latency (in ms)**					
		0.5	**1**	**2**	**4**	**6**	**8**
16 QAM	100	5.4	5.7	6.7	7.2	7.4	7.6
	200	5.7	6.7	7.2	7.6	7.8	–
4096QAM	100	0.9	3.8	4.2	4.5	–	–
	200	3.8	4.2	4.5	–	–	–

LDPC CODING

Binary LDPC codes have been known since the early 1960's but their capacity-approaching performance has been discovered only in the past decade. Currently there is intensive research activity to investigate deterministic constructions of such codes as well as to understand their theoretical limits [10]. It appears that high-rate LDPC codes with medium block length, whose parity-check matrices are constructed similarly to those of array codes, exhibit as good a performance as that of random LDPC codes. The array codes are two-dimensional codes that originally were proposed for detecting and correcting burst errors [11]. Array-code-based LDPC coding [12] was shown in [13] to offer a number of additional advantages for DSL transmission. In particular, the LPDC parity-check matrix is specified by a small set of parameters and constructed deterministically without requiring "preprocessing" operations. With the form of the array-code based parity-check matrix introduced in [13], encoding is achieved directly from the parity-check matrix and is linear in time.

Fig. 5 shows the SER performance of three LDPC codes of lengths N = 529, 2209, and 4489 and code rates 0.870, 0.915, and 0.940, respectively. In these simulations, the sum-product algorithm is employed with the number of iterations limited to 20.



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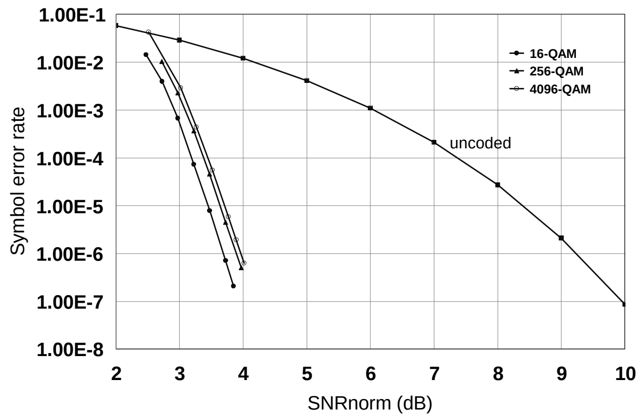

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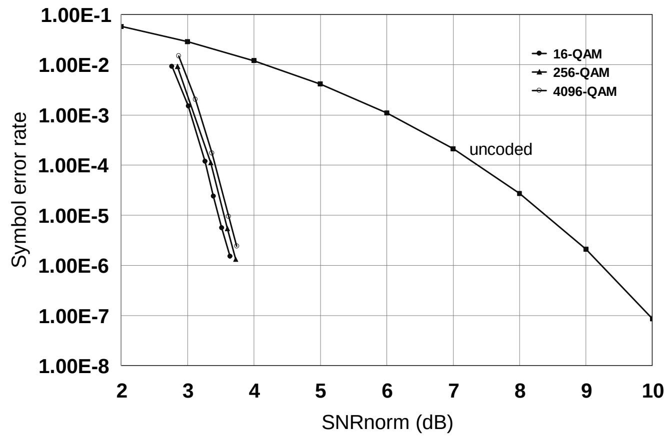

(c)

Fig. 5. Performance of three LDPC codes, namely (a) (529, 460), (b) (2209, 2021), and (c) (4489, 4221), for transmission over the AWGN channel using 16, 256, and 4096-QAM.

The results summarized in Table 2 show the net coding gains in dB at a SER of 10^{-7} for different values of coding latency (no outer RS code). Simulation results for codes longer than 7200 bits have not been obtained. Finally, the code rates were chosen in the range of 0.82 to 0.95. It can be seen that good coding gains can be achieved even for very tight latency constraints.

Table 2. Net coding gains in dB, and corresponding rate in bit per symbol, achieved by selected LDPC codes as a function of latency.

Modu-lation	**# of sub-carriers**	**Latency (in ms)**					
		0.5	**1**	**2**	**4**	**6**	**8**
16 QAM	100	4.7 dB (3.31 b/symb)	5.0 dB (3.54 b/symb)	5.4 dB (3.69 b/symb)	6.2 dB (3.71 b/symb)	6.3 dB (3.76 b/symb)	6.4 dB (3.79 b/symb)
	200	5.0 dB (3.54 b/symb)	5.4 dB (3.69 b/symb)	6.2 dB (3.71 b/symb)	6.4 dB (3.79 b/symb)	–	–
4096 QAM	100	4.3 dB (11.13 b/symb)	4.8 dB (11.45 b/symb)	5.4 dB (11.60 b/symb)	6.1 dB (11.65 b/symb)	6.3 dB (11.63 b/symb)	–
	200	4.8 dB (11.45 b/symb)	5.4 dB (11.60 b/symb)	6.1 dB (11.65 b/symb)	–	–	–

The TCM scheme defined in the ADSL standards achieves a net coding gain of 4.4 and 4.2 dB for 16-QAM and 4096-QAM, respectively, at a SER of 10^{-7}. The average rate is equal to 3.5 bits per 2D symbol for 16-QAM and 11.5 bits per 2D symbol for 4096-QAM. Note that the turbo codes, LDPC codes and the TCM scheme considered do not necessarily operate at the same rates. For TCM, the encoding and decoding operations extend over one frame period each, hence latency amounts to 0.5 ms. For this latency value, we see from Table 1 that the simulated turbo-coding scheme provides a coding gain of up to 5.7 and 3.8 dB for 16-QAM and 4096-QAM, respectively. The modest coding gain of 0.9 dB obtained for a large QAM constellation size and short interleaver length can be significantly improved, as mentioned earlier, by using a combination of coded and uncoded bits for symbol mapping. Once the latency restriction is relaxed, turbo codes clearly outperform TCM in all cases. For the latency value of 0.5 ms, we see from Table 2 that LDPC coding provides a coding gain of up to 5.0 dB and 4.8 dB for 16-QAM and 4096-QAM, respectively. Here, unlike in the turbo-coding case, uncoded bits were used for 4096-QAM, and therefore good coding gains are achieved also in this case. By increasing the code-word length to encompass more than one DMT frame, additional coding gains of 2 dB and more are possible using LDPC coding with respect to TCM. Note that the coding gain advantage of the turbo codes and the LDPC codes is even higher when lower error-rate values are considered because of the rather steep error-rate versus SNR characteristics for this code type. The gain for turbo coding can be further improved by increasing the interleaver length or the number of states for each component code (from the 8 states presented in this paper to 16 or 32 states).

We do not attempt to provide a one-to-one performance comparison of turbo and LDPC coding for DSLs because a fair comparison would require a number of parameters, such as code rate, latency, and implementation complexity, to be kept equal. We note however that code rate and coding gain can often be traded off for DSLs through the bit-loading process. That is, for a specified minimum operating margin, a particular data rate that can be achieved with a specific code-rate/coding-gain combination can also be achieved, for example, by increasing the code rate and decreasing the coding gain, or vice versa.

V. COMPLEXITY

For encoding, it can be assumed that computational complexity is essentially identical for turbo coding and for TCM. For LDPC coding, if the code word is obtained by multiplying the information block with the generator matrix of the code, encoding requires $\mathcal{O}(N^2)$ operations, where N is the length of the code. However, the family of LDPC codes proposed in [13] and used in the preceding section enjoys the desirable property of linear-time encodability, where encoding requires $\mathcal{O}(N)$ operations. Indeed, it was shown for this case that the complexity of LDPC encoding typically amounts to three times the encoding complexity of TCM.

As mentioned, there are two basic algorithms for decoding turbo and LDPC codes: The BCJR algorithm and the sum-product or belief propagation algorithm, respectively. To minimize the number of multiplications in a practical implementation, it is advantageous for both algorithms to compute and propagate messages that represent log-likelihood ratios. In both cases, simplified algorithms exist that aim at lowering implementation complexity at the price of some loss in performance. A comparison of the complexity of the various decoding algorithms would, however, exceed the scope of this paper, but a generally accepted fact is that LDPC decoding by the sum-product algorithm is computationally less complex than turbo decoding by the BCJR algorithm.

Finally, it should be mentioned that turbo- and LDPC-coding techniques may have stringent memory requirements, especially for long codes. This is an important aspect in the design of DSL transceivers.

VI. CONCLUSIONS

Multilevel modulation is essential in many communication systems to maximize the rate of information transfer under strict constraints on the transmit signal bandwidth. In this context, the symbol-mapping technique used to obtain coded multilevel transmit symbols plays a crucial role, as it affects both the system performance and the complexity of implementation. The technique based on set-partitioning of the transmit symbol set and double Gray-code labeling described in this paper is generic and can be employed in connection with a variety of capacity-approaching codes, such as turbo, turbo-like, or LDPC codes.

Capacity-approaching coding techniques can provide additional coding gains compared with the coding schemes used in current ADSL standards. It was shown that this coding gain is a valuable resource for increasing data rate and/or loop reach, which can be instrumental for an optimum usage of the local loop and the widespread deployment of ADSL services. Two practical approaches based on turbo and LDPC coding were presented. The possibility of incorporating both turbo- and LDPC-coding capability into the ADSL systems, a potentially interesting approach, was also discussed. It was not attempted to provide a one-to-one comparison of these two approaches because code parameters, encoding and decoding complexity, as well as other factors are in general different. The main conclusion is that both techniques appear to be practical for implementation, with a reasonable increase in transmitter/receiver complexity. It is expected that capacity-approaching coding techniques, such as those described in this paper, will soon find their way into future generations of DSL modems and cable transmission systems in general.

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