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Research Report

Properties of UWB 2PPM Transceiver with Noncoherent Receiver

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Properties of UWB 2PPM Transceiver System with Noncoherent Receiver

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1 Introduction

This report describes properties relevant for the implementation of the energy collecting receiver (ECR). It starts with the description of the system model comprising a 2PPM (2-slot pulse position modulation) transmitter, an UWB (ultra-wideband) channel and a noncoherent or energy collecting receiver.

Narrowband interference (NBI) can severely deteriorate the sensitivity of an energy collecting receiver as there is no mechanism like, e.g., matched filtering, that would distinguish to a certain degree between interfering and desired signal components. Even if the assumption is justified that NBI is a priori out of band, its expected intensity must be considered to design the slopes of the receiver filter. In Section 3 the effect of narrowband interference on the ECR's performance is assessed and compared with that of the coherent ideal matched filter receiver. The criterion for the comparison is the ratio of the mean value to the variance of the decision variable. It turns out that, for a practical set of system parameters, the ECR's sensitivity to NBI is about 15 dB higher than for the coherent matched filter receiver.

The considered noncoherent receiver performs a square operation along it's signal path, which results in spectral components with twice the maximum frequency of the received signal. Typically, a receiver circuit will cut-off some of these high frequency components. The effect of this energy loss is estimated in Section 4, the result is that even a cut-off frequency below the minimum signal frequency will only marginally deteriorate the receiver performance.

Section 5 shows the parallelism between the considered communication system to BPSK (binary phase shift keying) signaling over the discrete time memoryless AWGN channel; for this class of systems, some exemplary codes and their coding gains are listed.

2 System Model

2.1 Transmitter

The transmitter modulates the symbol sequence $\langle a_k \rangle$ with, $a_k \in \{0, 1\}$, such that each symbol determines the position of one UWB pulse. The shape of an individual pulse is defined by $g(t)$, which is the impulse response of an ideal bandpass filter with center frequency f_0 and bandwidth B. The pulse $g(t)$ has energy $2B$, i.e., $\|g\|^2 = \int_{-\infty}^{\infty} g^2(t)\, dt = 2B$. The choice of an ideal bandpass filter $g(t)$ is justified, because in contrast to an implementable but more complex filter, the ideal bandpass filter results in simpler analytical expressions for the signals involved in the receiver. However, we observe that the receiver characteristics will change within some range when another, more realistic, bandpass filter is assumed, see [1]. The transmitted signal is of the form

$$u(t) = \sqrt{\frac{E_t}{2B}} \sum_{k=0}^{K-1} c_k\, g(t - kT - a_k \Delta_T), \tag{1}$$

representing a data block of K data symbols. The time interval available for the transmission of an individual symbol is T; the corresponding data symbol a_k determines whether the pulse is transmitted at the beginning of this interval or with a time offset Δ_T. The energy per transmitted pulse is E_t. The sequence $\langle c_k \rangle,\ c_k \in \{-1, +1\}$, is an i.i.d. pseudo-random binary sequence that randomizes the polarity of the transmitted pulses to smooth the power spectrum of the signal $u(t)$; thus the power spectral density of the transmitted signal is proportional to the energy density spectrum of the transmitted pulse. Reference [2] documents properties of a signal with a power spectrum that is smooth in the sense of the FCC's emission rules [3]. From this it follows that for a symbol rate, higher than about 10^6 symbols/s, the power limit of -41.25 dBm/MHz cannot be exploited, if no polarity randomization is employed. The sequence $\langle c_k \rangle$ has no impact on the receiver's design or performance as we consider noncoherent receivers.

To prevent intersymbol interference and to maintain the orthogonality of the received symbols, it is required that the delay Δ_T as well as $T - \Delta_T$ exceed the maximum channel delay spread τ_c. Note that this condition limits the maximum data rate to $1/(2\tau_c)$ for 2PPM. Let $b(t)$ be the received pulse shape representing the combined response of transmitter filter $g(t)$, transmitter antenna, propagation channel, and receiver antenna [4]. Note that because of the wide bandwidth, the received pulse shape is not only influenced by the transmitter filter and the channel impulse response but also by the transmitter's and receiver's amplifier and antenna. For this and various other reasons, channel models for the UWB channel include the characteristics of the transmitter and receiver antennas. This composite channel impulse response corresponds to our received pulse shape $b(t)$, as we assume both the signal at the transmitter antenna feedpoint and the impulse response of the receiver filter to be ideal bandpass impulse responses. For this reason we can call the *received pulse shape*, $b(t)$, synonymously *channel impulse response*. The signal that appears at the feed point of the receiver antenna and that corresponds to the symbol a_k is

$$r(a_k, t) = c_k b(t - kT - a_k \Delta_T). \tag{2}$$

The energy per received pulse is

$$E_r = \int_0^{\Delta_T} b^2(t)\, dt \tag{3}$$

and we define the reciprocal value of the path loss or path gain, α, as the ratio of the received and transmitted energies, i.e.,

$$\alpha = \frac{E_r}{E_t}. \tag{4}$$

2.2 The Receiver

The optimal receiver in the absence of channel state information is the generalized maximum likelihood receiver (GMLR) derived in [5] and described by the decision rule

$$z_k \underset{\hat{a}_k=0}{\overset{\hat{a}_k=1}{\gtrless}} 0, \tag{5}$$

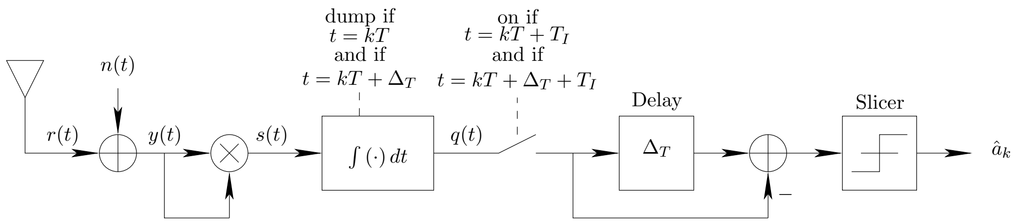


Figure 1: An architecture of the generalized maximum likelihood receiver for 2PPM signals.

with the definition of the decision variable for the k-th symbol

$$z_k = \int_{kT}^{kT+T_I} y^2(t)\, dt \quad - \int_{kT+\Delta_T}^{kT+\Delta_T+T_I} y^2(t)\, dt, \tag{6}$$

where $T_I < \Delta_T$ is a variable integration duration that can be adapted to the channels delay spread, τ_c. A possible implementation of this receiver is shown in Fig. 2.2 The bit error probability (BEP) of the GMLR is expressed by the approximation [1]

$$P_e = \frac{1}{2}\text{erfc}\left(\frac{\eta(T_I)E_r/N_0}{2\sqrt{T_I B + \eta(T_I)E_r/N_0}}\right), \tag{7}$$

where $\eta(T_I)$ is the ratio of the captured energy per received pulse to the total energy of the received pulse E_r, i.e.,

$$\eta(T_I) = \frac{\int_0^{T_I} b^2(t)\, dt}{\int_0^{\Delta_T} b^2(t)\, dt}. \tag{8}$$

For reference we also give the BEP of the coherent maximum likelihood receiver (MLR), which is

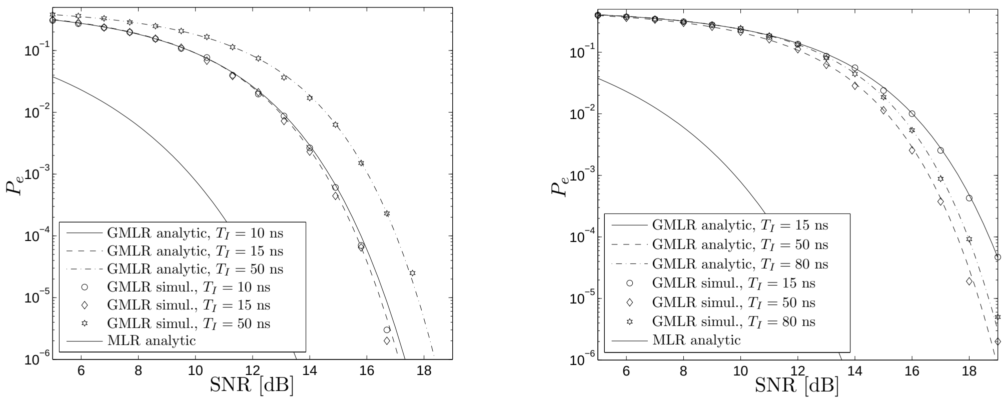
$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_r}{2N_0}}\right), \tag{9}$$

for the case where the pulse polarity is constant, i.e., $c_k = 1$ for all k; note that the performance of the corresponding MLR is reduced if the pulse polarity is randomly chosen.

The BEP over SNR curve of the GMLR and the MLR is shown in Fig. 2 for a channel impulse response realization of the channel model CM1 and CM4 respectively, these channel models are defined in [4].

3 Effect of Narrowband Interference

Narrowband interference reduces the sensitivity of a receiver, which is equivalent to increasing the BEP for a given receiver signal-to-noise ratio (SNR). As an indirect measure for this effect, we derive the ratio of the mean value of the decision variable to its standard



(a) For channel impulse response 1 of model CM1.

(b) For channel impulse response 1 of model CM4.

Figure 2: BEP of GMLR for various integration durations T_I and BEP of the MLR.

deviation under the assumption of narrowband interference. For comparison we derive this ratio for both the GMLR receiver and the MLR for 2PPM signals. Note that this ratio is proportional to the argument of the erfc(·) functions that describe the BEP of the corresponding receivers.

3.1 Decision Variable Statistics for the GMLR

We start by deriving the statistics of the decision variables z_k, under the assumption of narrowband interference plus noise. As the decision variables are i.i.d. random variables, we consider only the variable z_0 which is denoted by z for simplicity. In (6) the decision variable z is defined as the difference $z = f_s - g_s$ of the integrals

$$f_s = \int_0^{T_I} [b(t) + u(t) + n(t)]^2 \, dt, \tag{10}$$

$$g_s = \int_0^{T_I} [u(t + \Delta_T) + n(t + \Delta_T)]^2 \, dt, \tag{11}$$

which are the integrals of the squared observed signal $y(t) = r(t) + u(t) + n(t)$ in the first and the second part of a symbol interval, where $u(t) = \sqrt{2P_u} \cos(2\pi f_0 t + \varphi_0)$, with power P_u. To prepare the derivation of the mean and variance of these two terms we summarize some results from an intermediate calculation [6]:

$$y = 2 \int_0^{T_I} [x(t) + n(t)]^2 \, dt \approx \sum_{n=0}^{N_\Delta - 1} |x_n + n_n|^2,$$

where x_n and n_n are the corresponding discrete time complex baseband signals of $x(t)$ and $n(t)$. The distribution of y is called a non-central chi-square distribution

$$\frac{1}{2N_0}\left(\frac{y}{s^2}\right)\frac{N_\Delta - 1}{2} e^{-\frac{s^2-y}{2N_0}} I_{N_\Delta-1}\left(\sqrt{y}\frac{s}{N_0}\right),$$

with

$$s^2 = \sum_{n=0}^{N_\Delta-1} |x_n|^2 \approx 2\int_0^{T_I} x^2(t)\,dt,$$

and the degree of freedom

$$2N_\Delta = 2T_I B.$$

The mean value of y is

$$\mu_y = 2N_\Delta N_0 + s^2 \approx 2N_0 T_I B + 2\int_0^{T_I} x^2(t)\,dt,$$

and the variance is

$$\sigma_y^2 = 4N_\Delta N_0^2 + 4N_0 s^2 \approx 4N_0 T_I B + 8N_0\int_0^{T_I} x^2(t)\,dt.$$

Using these results and assuming that $u(t)$ and $b(t)$ are deterministic signals we observe that the sampled values f_s and g_s are both non-central chi-square distributed random variables with degree of freedom $2N_\Delta$. f_s and g_s are statistically independent with respect to the noise $n(t)$ because of the time shift Δ_T, which is present in (10) but not in (11). Their mean and variance are

$$\mu_{f_s} = N_0 T_I B + \int_0^{T_I} [b(t)+u(t)]^2\,dt,$$

$$\sigma_{f_s}^2 = N_0^2 T_I B + 2N_0\int_0^{T_I} [b(t)+u(t)]^2\,dt, \tag{12}$$

$$\mu_{g_s} = N_0 T_I B + \int_0^{T_I} u^2(t+\Delta_T)\,dt,$$

and

$$\sigma_{g_s}^2 = N_0^2 T_I B + 2N_0\int_0^{T_I} u^2(t+\Delta_T)\,dt. \tag{13}$$

The decision variable $z = f_s - g_s$ has the mean value

$$\begin{aligned}\mu_z &= \mu_{f_s} - \mu_{g_s}\\ &= \int_0^{T_I} b^2(t)\,dt + 2\int_0^{T_I} b(t)u(t)\,dt + \int_0^{T_I} u^2(t)\,dt - \int_0^{T_I} u^2(t+\Delta_T)\,dt.\end{aligned} \tag{14}$$

Considering the received pulse shape $b(t)$ and the interference signal $u(t)$ again as realizations of random signals, then μ_z has also a mean and a variance. Evaluation of the first term of (14) yields

$$\int_0^{T_I} b^2(t)dt = E_r\eta(T_I), \tag{15}$$

see [1]. The second term of (14) is described as a random variable by (48) in Appendix A, i.e.,

$$2\int_0^{T_I} b(t)u(t)\,dt = 2\rho_{u,b} \sim \mathcal{N}\left(0, \frac{2P_uE_r\eta(T_I)}{B}\right). \tag{16}$$

The third therm can be written by inserting the definition of $u(t)$:

$$\begin{aligned}
\int_0^{T_I} u^2(t)\,dt &= 2P_u\int_0^{T_I}\cos^2(2\pi f_0t+\varphi_0)\,dt \\
&= P_u\int_0^{T_I} 1+\cos(4\pi f_0t+2\varphi_0)\,dt \\
&= P_uT_I + \frac{P_u}{4\pi f_0}\sin(4\pi f_0t+2\varphi_0)\Big|_0^{T_I} \\
&= P_uT_I + \frac{P_u}{4\pi f_0}[\sin(4\pi f_0T_I+2\varphi_0)-\sin(2\varphi_0)] \\
&= P_uT_I + \frac{P_u}{2\pi f_0}\sin(2\pi f_0T_I)\cos(2\pi f_0T_I+2\varphi_0). \qquad (17)
\end{aligned}$$

Similarly, the fourth term yields

$$\begin{aligned}
\int_0^{T_I} u^2(t+\Delta_T)\,dt &= 2P_u\int_0^{T_I}\cos^2(2\pi f_0(t+\Delta_T)+\varphi_0)\,dt \\
&= 2P_u\int_{\Delta_T}^{T_I+\Delta_T}\cos^2(2\pi f_0t+\varphi_0)\,dt \\
&= P_uT_I + \frac{P_u}{4\pi f_0}\sin(4\pi f_0t+2\varphi_0)\Big|_{\Delta_T}^{T_I+\Delta_T} \\
&= P_uT_I + \frac{P_u}{4\pi f_0}[\sin(4\pi f_0(T_I+\Delta_T)+2\varphi_0)-\sin(4\pi f_0\Delta_T+2\varphi_0)] \\
&= P_uT_I + \frac{P_u}{2\pi f_0}\sin(2\pi f_0T_I)\cos(4\pi f_0\Delta_T+2\pi f_0T_I+2\varphi_0). \qquad (18)
\end{aligned}$$

With this, the difference of (17) and (18) is

$$\begin{aligned}
\int_0^{T_I} u^2(t)\,dt - \int_0^{T_I} u^2(t+\Delta_T)\,dt &= \frac{P_u}{4\pi f_0}\sin(2\pi f_0T_I) \\
&\quad [\cos(2\pi f_0T_I+2\varphi_0)-\cos(4\pi f_0\Delta_T+2\pi f_0T_I+2\varphi_0)] \\
&= \frac{P_u}{2\pi f_0}\sin(2\pi f_0T_I)\sin(2\pi f_0\Delta_T)\sin(2\pi f_0(\Delta_T+T_I)+2\varphi_0).
\end{aligned} \tag{19}$$

A practical narrowband interference signal $u(t)$ will be modulated, therefore we assume that the phase φ_0 is randomly distributed between 0 and 2π, which means that the difference (19) oscillates between $\pm\frac{P_u}{2\pi f_0}$ in the worst case, i.e., when the product $\sin(2\pi f_0 T_I)\sin(2\pi f_0 \Delta_T)$ equals one. In this case the difference (19) has zero mean and variance $\frac{1}{2}[P_u/(2\pi f_0)]^2$.

With this result and with (17) and (18) we can write the mean and variance of μ_z as

$$\mu_{\mu_z} = E_r \eta(T_I), \tag{20}$$

and

$$\sigma^2_{\mu_z} = \frac{2P_u E_r \eta(T_I)}{B} + \frac{P_u^2}{8\pi^2 f_0^2}, \tag{21}$$

where we assumed that $\rho_{u,b}$ is statistically independent form (17) and (18); note that $\rho_{u,b}$ is a function of both random signals, U(t) and B(t).

The variance $\sigma^2_{f_s}$ given in (12) can be expanded to

$$\sigma^2_{f_s} = N_0^2 T_I B + 2N_0 \int_0^{T_I} b^2(t)\,dt + 4N_0 \int_0^{T_I} b(t)u(t)\,dt + 2N_0 \int_0^{T_I} u^2(t)\,dt. \tag{22}$$

With (15), (16) and (17) this becomes

$$\sigma^2_{f_s} = N_0^2 T_I B + 2N_0 E_r \eta(T_I) + 4N_0 \rho_{ub} + 2N_0 P_u T_I, \tag{23}$$

where we ignored the second term in (17) which is much smaller than the part $P_u T_I$ for practical cases where $1/f_0 \ll T_I$. The variance $\sigma^2_{g_s}$ given by (13) becomes with (18)

$$\begin{aligned} \sigma^2_{g_s} &= N_0^2 T_I B + 2N_0 \int_0^{T_I} u^2(t+\Delta_T)\,dt \\ &= N_0^2 T_I B + 2N_0 P_u T_I, \end{aligned} \tag{24}$$

where we ignored the second term in (18) for the same reason as above.

The variance σ^2_z of the decision variable $z = f_s - g_s$ is a function of $\sigma^2_{\mu_z}$, $\sigma^2_{f_s}$ and $\sigma^2_{g_s}$. The component $4N_0\rho_{ub}$ of $\sigma^2_{f_s}$, see (23), is a random variance and therefore complicates the computation of σ^2_z. However, for the practical range of parameters, $T_I > 10/B$, and $E_r > 10N_0$, the standard deviation of this term is dominated with a certain probability by the terms $2P_u E_r \eta(T_I)/B$ and $2N_0 E_r \eta(T_I)$ in (21) and (23). For $P_u/B \leq N_0$ the term $2N_0 E_r \eta(T_I)$ is dominant over the standard deviation $\sqrt{8N_0^2 E_r P_u \eta(T_I)/B}$ of the term $4N_0\rho_{ub}$, because

$$\frac{\sigma_3^2}{\sigma_{\sigma_4}} = \frac{2N_0 E_r \eta(T_I)}{\sqrt{8}N_0\sqrt{P_u/B}\sqrt{E_r\eta(T_I)}} = \frac{\sqrt{E_r\eta(T_I)}}{\sqrt{2P_u}/B} > \frac{\sqrt{10N_0}}{\sqrt{2N_0}} = \sqrt{5},$$

whereas for the case $P_u/B > N_0$ the term $2P_u E_r \eta(T_I)/B$ is dominant over the standard deviation $\sqrt{8N_0^2 E_r P_u \eta(T_I)/B}$:

$$\frac{\sigma^2_{\mu_2}}{\sigma_{\sigma_4}} = \frac{2(P_u/B)E_r\eta(T_I)}{\sqrt{8}N_0\sqrt{P_u/B}\sqrt{E_r\eta(T_I)}} = \frac{\sqrt{P_u/B}\sqrt{E_r\eta(T_I)}}{\sqrt{2}N_0} > \frac{\sqrt{N_0}\sqrt{10N_0}}{\sqrt{2}N_0} = \sqrt{5}$$

From this argumentation it follows that ignoring the term $4N_0\rho_{ub}$ in (23), results in at least a rough approximation of $\sigma^2_{f_s}$.

We can now give an expression for the variance, σ_z^2, of the decision variable z. As noted above, the terms $\sigma^2_{f_s}$ and $\sigma^2_{g_s}$ stem from the noise signal $n(t)$ within the disjoint intervals $[kT, kT+\Delta_T]$ and $[kT+\Delta_T, kT+T]$ and are therefore statistically independent. The term $\sigma^2_{\mu_z}$ stems from the random channel $\mathrm{B}(t)$ and the random phase φ_0 of the interference $u(t)$, hence the variability of z described by $\sigma^2_{\mu_z}$ is statistically independent from the noise. With this and by skipping the term $4N_0\rho_{ub}$, the total variance of the decision variable z is

$$\begin{aligned}\sigma_z^2 &= \sigma^2_{\mu_z} + \sigma^2_{f_s} + \sigma^2_{g_s} \\ &= \frac{2P_uE_r\eta(T_I)}{B} + \frac{P_u^2}{8\pi^2 f_0^2} + 4N_0P_uT_I + 2N_0^2T_IB + 2N_0E_r\eta(T_I). \end{aligned} \tag{25}$$

The mean value of z is

$$\mu_{\mu_z} = E_r\eta(T_I). \tag{26}$$

Hence, the ratio of the mean value of z to the standard deviation of z for the GMLR is

$$R_{\mathrm{GMLR}} = \frac{E_r\eta(T_I)}{\sqrt{\frac{2P_uE_r\eta(T_I)}{B} + \frac{P_u^2}{8\pi^2 f_0^2} + 4N_0P_uT_I + 2N_0^2T_IB + 2N_0E_r\eta(T_I)}}. \tag{27}$$

For an integration durations in the order of $T_I = 40$ ns this analytical expression has been tested by simulation which showed good agreement. Note that for $P_u = 0$, i.e., when no narrowband interference is present, then the BEP $P_e = P(z < 0)$ is approximated by

$$P_e = \frac{1}{2}\mathrm{erfc}\left(\frac{1}{\sqrt{2}}R_{\mathrm{GMLR}}\right),$$

compare with (7).

3.2 Decision Variable Statistics for the MLR

The MLR for 2PPM signals bases the symbol decision, $\hat{a}_0$, on the variable $z = f_s - g_s$. For the MLR which is a coherent receiver, we assume that the polarity c_k in (1) is always positive, i.e., $c_k = 1$ for all k; for this type of receiver, a random polarity would result in a reduced sensitivity. Unlike the GMLR which computes the correlation of the received signals with themselves, the MLR correlates the received signals with the template $b(t)$; i.e., under the assumption that $a_k = 0$ the correlator samples are

$$f_s = \int_0^{\Delta_T} b(t)[b(t) + n(t) + u(t)]\, dt, \tag{28}$$

$$g_s = \int_0^{\Delta_T} b(t)[n(t+\Delta_T) + u(t+\Delta_T)]\, dt. \tag{29}$$

Here f_s corresponds to the first sample taken during the symbol interval and g_s corresponds to the second sample. Note that the coherent receiver captures the energy of the received pulse within an interval of duration Δ_T, while the optimum integration interval for the noncoherent receiver is of the reduced duration T_I, with $T_I < \Delta_T$. The only component of these samples, whose statistics is not yet described above, is the term

$$a := \int_0^{\Delta_T} b(t)n(t)\,dt$$

of which statistically independent realizations appear in both terms, f_s and g_s. The mean value of a is zero, because $n(t)$ has zero mean; the variance is computed by the assumption that $n(t)$ is white Gaussian noise, i.e., has the autocorrelation function $(N_0/2)\delta(\tau)$. This assumption does not change the result, because the output of the matched filter, which is implicitly contained in the MLR, does not depend on whether the bandwidth of $n(t)$ is limited to the bandwidth of the matched filter impulse response $b(t)$ or is unlimited. Hence, the variance of a is

$$\begin{aligned}
\sigma_a^2 &= \mathbb{E}\left\{\left[\int_0^{\Delta_T} b(t)n(t)\,dt\right]^2\right\} \\
&= \int_0^{\Delta_T}\int_0^{\Delta_T} b(t)b(\tau)\mathbb{E}\left\{n(t)n(\tau)\right\}\,dtd\tau \\
&= \frac{N_0}{2}\int_0^{\Delta_T}\int_0^{\Delta_T} b(t)b(\tau)\delta(t-\tau)\,dt\,d\tau \\
&= \frac{N_0}{2}\int_0^{\Delta_T} b^2(t)\,dt \\
&= \frac{N_0 E_r}{2}.
\end{aligned} \tag{30}$$

The terms

$$\int_0^{\Delta_T} b(t)\,u(t)\,dt \qquad \text{and} \qquad \int_0^{\Delta_T} b(t+\Delta_T)u(t+\Delta_T)\,dt,$$

have variance $E_r\eta(T_I)P_u/B$ as characterized by (48) and are statistically independent for the assumption that the phases φ_0 of the interference $u(t)$ in the two integration intervals are statistically independent; this assumption is justified when we assume that the narrowband interference $u(t)$ is a modulated signal. With this, (30), (15), (48), and from $\eta(\Delta_T) = 1$, see (8), f_s is characterized by

$$f_s \sim \mathcal{N}\left(E_r, \frac{N_0 E_R}{2} + \frac{P_u E_r}{2B}\right). \tag{31}$$

and g_s is described by

$$g_s \sim \mathcal{N}\left(0, \frac{N_0 E_R}{2} + \frac{P_u E_r}{2B}\right). \tag{32}$$

As the components of f_s are statistically independent of the components of g_s, the decision variable z is characterized by

$$z = f_s - g_s \sim \mathcal{N}\left(E_r, N_0 E_R + \frac{P_u E_r}{B}\right). \tag{33}$$

In the considered case where $a_k = 0$, a decision error occurs if $z < 0$. As the 2PPM scheme is symmetric with respect to $a_k = 0$ and $a_k = 1$, the BEP is $P_e = P(z < 0)$ and is given by

$$\frac{1}{2}\mathrm{erfc}\left(\frac{1}{\sqrt{2}} R_{\mathrm{MLR}}\right),$$

with

$$R_{\mathrm{MLR}} = \frac{E_r}{\sqrt{N_0 E_r + \frac{P_u E_r}{B}}}. \tag{34}$$

In the absence of narrowband interference, i.e., when $P_e = 0$, the BEP is

$$P_e = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_r}{2N_0}}\right). \tag{35}$$

3.3 Comparison of GMLR and MLR

We make the following assumptions: $B = 1$ GHz, $f_0 = 4$ GHz, $T_I = 40$ ns, $\eta(T_I) = 0.8$, $P_u < E_r f_0$, $E_r = 10N_0$ (this corresponds to a receiver SNR of 10 dB. For the GMLR the contribution of the narrowband interference to the denominator of (27) is

$$P_u\left(\frac{2E_r\eta(TI)}{B} + \frac{P_u}{8\pi^2 f_0^2} + 4N_0 T_I\right) \approx P_u E_r \cdot 17.6 \cdot 10^{-9}\ \mathrm{s},$$

while for the MLR the contribution of the narrowband interference to the denominator of (34) is

$$\frac{P_u E_r}{B} = P_u E_r \cdot 0.5 \cdot 10^{-9}\ \mathrm{s}.$$

We observe, that the MLR has a better immunity to narrowband interference. One consequence for a practical GMLR receiver implementation with the given numerical parameters could be that the receiver filter, suppressing out-of-band interference, in contrast to the MLR needs higher attenuation by about 15 dB. This result is plausible because the GMLR uses the received signal as a correlation template and therefore collects any signal passing the receiver filter, while the MLR has a fixed receiver template $b(t)$ that attenuates signal components that are not in the signal space spanned by the template.

4 Effect of Finite Integrator Bandwidth

Ideally, the integrator will compute the integral of the signal $s^2(t)$ over the interval $[0, T_I]$, i.e, the integrator output is

$$q = \int_0^{T_I} s^2(t)\, dt.$$

In practice, the bandwidth of the square argument in the integration, $s^2(t)$, will be reduced in bandwidth. This effect is modelled by the convolution with the response of an ideal lowpass filter response $g(t)$ with one-sided bandwidth B and no attenuation in the passband, hence,

$$g(t) = \frac{4B\sin(2\pi Bt)}{2\pi Bt} \overset{t,f}{\circ\!\!-\!\!\bullet} \begin{cases} 1, & \text{for } |f| < B, \\ 0, & \text{else.} \end{cases}$$

We compute the integrator output as

$$q' = \int_0^{T_I} [s^2(t) * g(t)]\, dt.$$

To see the impact of the bandwidth B on the integrator output q' we compare this with the ideal integrator output q. The impulse response of the lowpass filter, $g(t)$, decays to zero in proportion to $1/t$, e.g., for $B = 2$ GHz, the response $g(t)$ decays to a tenth of its maximum value, $g(0)$, for $t_{10} = 10/(2\pi B) = 0.8$ ns. The support of the signal $y(t) = s^2(t) * g(t)$ is infinite. However, with $s^2(t)$ having finite support, and $g(t)$ having a decay time that is much shorter than the integration duration T_I, i.e, when $T_I >\gg t_{10}$, we can approximate:

$$q' = \int_0^{T_I} [s^2(t) * g(t)]\, dt \approx q' \int_{-\infty}^{\infty} [s^2(t) * g(t)]\, dt.$$

With

$$s^2(t) * g(t) \overset{t,f}{\circ\!\!-\!\!\bullet} [S(f) * S(f)]\text{rect}(f/B),$$

where rect(f) is 1 for $|f| < B$ and zero outside, we can write

$$\begin{aligned} q' \approx \int_{-\infty}^{\infty} s^2(t) * g(t)\, e^{-2\pi ft}\, dt\bigg|_{f=0} &= \int_{-\infty}^{\infty} [S(f) * S(f)]\text{rect}(f/B)\bigg|_{f=0} \\ &= [S(f) * S(f)]|_{f=0} \\ &= \int_{-\infty}^{\infty} S(f)S(-f)\, df \\ &= \int_{-\infty}^{\infty} S(f)S^*(f)\, df \\ &= \int_{-\infty}^{\infty} |S(f)|^2\, df. \end{aligned} \tag{36}$$

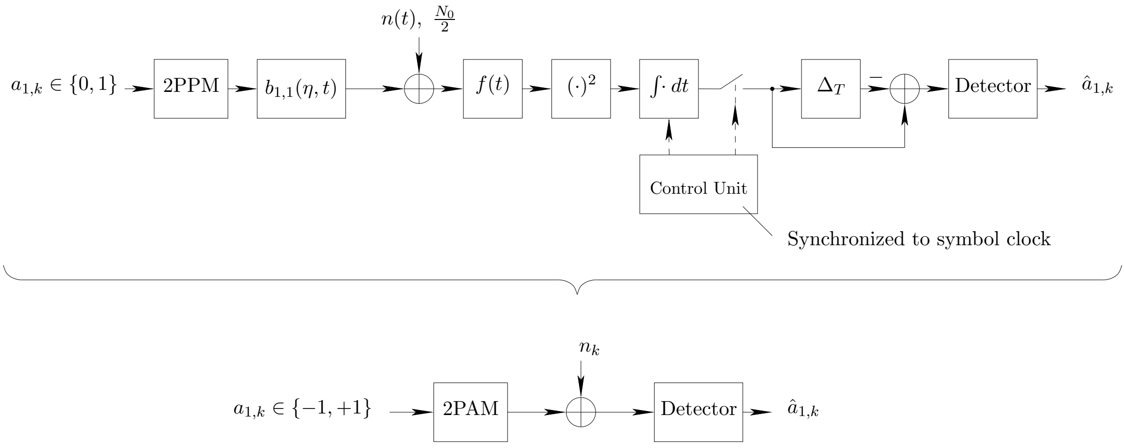
Note that

$$q = \int_0^{T_I} s^2(t)\, dt = \int_{-\infty}^{\infty} |S(f)|^2\, dt$$

i.e., we conclude from the above approximation that $q \approx q'$. This means, that a bandwidth $B = 2$ GHz of the lowpass filter has only a negligible impact on the receiver performance. An intuitive explanation of this effect is that the short time integration over the interval $[0, T_I]$ is equivalent to a lowpass filtering; thus, the effect of an additional lowpass filter changes the result only marginally.

5 Coding for a 2PPM System with an Energy Collecting Receiver

From the analysis in [1] it follows that the communication channel including the 2PPM transmitter, the wireless propagation channel and the noncoherent energy collecting receiver, can by approximation be modelled as a symbol-clocked discrete-time 2PAM transmitter, a memoryless AWGN channel and a coherent receiver. Figure 3 shows the real system and the simplified equivalent model. With this it follows that all codes that are



$$n_k \sim \mathcal{N}(\mu_n, \sigma_n^2)$$
$$\mu_n = \chi(t_1) - \chi(t_2)$$
$$\sigma_n^2 = \sigma_\psi^2(t_1) + \sigma_\psi^2(t_2) + \sigma_\omega^2(t_1) + \sigma_\omega^2(t_2) + 2\sigma_\zeta^2$$

Figure 3: 2PPM system with energy collecting receiver and simplified model.

suited for the discrete-time memoryless AWGN channel can also be applied to the considered UWB system. Table 1 gives some examples of codes and their coding gain for the raw bit error rates 10^{-2} and 10^{-3}.

6 Conclusion

This work reviews the BEP performance of a the noncoherent generalized maximum likelihood receiver (GMLR) in comparison with the BEP of the coherent maximum likelihood receiver (MLR).

As a measure for the sensitivity of the GMLR and the MLR to narrowband interference, we defined the ratio of the mean value of a decision variable to its standard deviation. It turned out that the MLR has better immunity to narrowband interference by about 15 dB compared to the GMLR. This is plausible as the GMLR uses the received signal as a correlation template and therefore collects any signal passing the receiver filter, while the MLR uses a perfect channel estimate as the correlation template, which attenuates signals that do not perfectly lie in the signal space spanned by the template.

Code	Uncoded BER	Coded BER	Source
Rate 1/2 convolutional code with constraint length 7 and hard decision Viterbi decoding	10^{-2} 10^{-3}	$2.8 \cdot 10^{-3}$ $1.8 \cdot 10^{-6}$	[7], Fig. 8-2-21
Rate 1/3 convolutional code with constraint length 7 and hard decision Viterbi decoding	10^{-2} 10^{-3}	10^{-3} $\ll 10^{-6}$	[7], Fig. 8-2-21
Hamming (15,11) code with hard decision decoding	10^{-2} 10^{-3}	$> 10^{-2}$ $3.5 \cdot 10^{-4}$	[8], Fig. 12-7
Hamming (15,11) code with soft decision decoding	10^{-2} 10^{-3}	$2.1 \cdot 10^{-2}$ $1.8 \cdot 10^{-5}$	[8], Fig. 12-7

Table 1: Examples of codes and their gain for an AWGN channel and binary antipodal modulation (BPSK).

The squared received signal contains second order intermodulation products which cause signal spectra that are nonzero for frequencies up to twice the maximum signal frequency. It was shown that spectral components above much above $1/T_I$ do not contribute to the decision variables. The reason for this is that the integrate and dump unit together with the sampler can be understood as a short time integrator, which obviously has a cut-off frequency proportional to the reciprocal of the integration duration T_I, this is typically a few tens of a nanosecond. This favorable property allows to use low-bandwidth circuits to process the squared signal, which fit well into the concept of a low complexity and low cost receiver.

The system considered in this work, consisting of a 2PPM transmitter and a noncoherent receiver results in approximately the same statistics of the decision variables as the well documented system of a BPSK (binary phase shift keying) transmitter and a coherent receiver. For the latter a large class of well documented codes exists that can be applied.

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A Response of a Correlator to a Cosine Signal

We assume a correlation template $b(t)$ which is multiplied with the received narrowband signal $u(t) = \sqrt{2P_u}\cos(2\pi f_I t + \varphi_0)$; this product is integrated over the interval $[0, T_I]$ and denoted as $\rho_{u,b}$, i.e.,

$$\rho_{u,b} = \int_0^{T_I} u(t)b(t)\,dt. \tag{37}$$

In the sequel we derive a statistical characterization of $\rho_{u,b}$ on basis of the statistical properties of the template $b(t)$ and the phase φ_0. For this purpose we define the normalized received pulse $\bar{b}(t)$ with energy $\|\bar{b}(t)\|^2 = 2B$; with this and according to Subsection 2.1 the received pulse is expressed as $b(t) = \sqrt{E_r/(2B)}\,\bar{b}(t)$. The Fourier transform of $\bar{b}(t)$ which considers only the integration interval $[0, T_I]$ is

$$\bar{B}(f) := \int_0^{T_I} \bar{b}(t)e^{-i2\pi ft}\,dt.$$

This representation suggests to interpret $\bar{B}(f)$ as the sum of many statistically independent random variables, i.e., the integral over the random received pulse shape weighted with an exponential function; from the CLR (central limit theorem) it follows that $\bar{B}(f)$ is for any f a complex Gaussian distributed random variable. As the real and imaginary parts of the complex function $\cos(2\pi f_0 t) - i\sin(2\pi f_0 t) = e^{-i2\pi f_0 t}$ are orthogonal, it follows that the imaginary and the real part of $\bar{B}(f)$ are statistically independent. Note that $\bar{b}(t)$ is assumed to be bandlimited to the frequency interval $[f_0 - B/2, f_0 + B/2]$, the spectrum $\bar{B}(f)$, however, is not bandlimited because the considered interval $[0, T_I]$ represents a limitation on the time axis. However, for practical bandwidths B on the order of 1 GHz, and integration durations T_I on the order of several tens of a ns, the time bandwidth product $T_I B \gg 1$. Hence, we can assume that the spectrum $\bar{B}(f)$ is nonzero only for $f \in [f_0 - B/2, f_0 + B/2]$.

The energy of $\bar{B}(f)$ is identical to the fraction of the energy of $\bar{b}(t)$, which falls into the interval $[0, T_I]$, i.e.,

$$\begin{aligned}\int_{-\infty}^{\infty} |\bar{B}(f)|^2\,df &= \int_0^{T_I} |\bar{b}(t)|^2\,dt \\ &= 2B\,\eta(T_I);\end{aligned} \tag{38}$$

the definition of $\eta(T_I)$ is given in (8). We assume further that for an integration interval shorter than the absolute channel delay spread τ_c, the number of echoes that fall into the integration interval is large enough such that $\bar{B}(f)$ can still be assumed to be Gaussian distributed for any $f \in [f_0 - B/2, f_0 + B/2]$.

We assume that within this frequency interval, the variance of $\bar{B}(f)$, i.e., $\mathbb{E}\left\{|\bar{B}(f)|^2\right\}$

does not depend on f. Based on this and with (38), which is a constant, we can write

$$\begin{aligned}\int_{f_0-B/2}^{f_0+B/2} |\bar{B}(f)|^2\, df &= \mathbb{E}\left\{\int_{f_0-B/2}^{f_0+B/2} |\bar{B}(f)|^2\, df\right\} \\ &= \int_{f_0-B/2}^{f_0+B/2} \mathbb{E}\left\{|\bar{B}(f)|^2\right\}\, df \\ &= 2B\mathbb{E}\left\{|\bar{B}(f)|^2\right\} \\ &= 2B\eta(T_I); \end{aligned} \tag{39}$$

this implies that $\mathbb{E}\left\{|\bar{B}(f)|^2\right\} = \eta(T_I)$, i.e., that

$$\Re\left\{\bar{B}(f)\right\}, \Im\left\{\bar{B}(f)\right\} \sim \mathcal{N}(0, \eta(T_I)/2). \tag{40}$$

We denote the complex baseband transform of $\bar{B}(f)$ as $\bar{B}_l(f)$, with real and imaginary part $\bar{B}_{r,l}(f)$ and $\bar{B}_{i,l}(f)$, respectively. From the definition of the equivalent baseband transform in [7] it follows that these components are i.i.d. for any f and Gaussian distributed [7]. With (40) and because the energy of the signal is half the energy of the equivalent signal in baseband representation, we conclude that the variance of these terms is $2\eta(TI)$, hence

$$\bar{B}_{i,l}(f),\ \bar{B}_{r,l}(f) \sim \mathcal{N}(0, 2\eta(T_I)). \tag{41}$$

Note further that in practice the average power spectral density $\mathbb{E}\left\{|B(f)|^2\right\}$ decays with $1/f^2$; we ignore this effect for simplicity and obtain an approximation that is the more accurate; the lower the relative signal bandwidth B/f_0 is. In practice, the absolute bandwidth will be on the order of 1GHz, and the center frequency is within the interval $[3.35 - 10.35]$ GHz

To derive the distribution of $\rho_{u,b}$, we express (37) as a function of $B(f)$:

$$\begin{aligned}\rho_{u,b} &= \sqrt{\frac{E_r}{2B}} \int_{t-T_I}^{t} u(t)\bar{b}(t)\, dt \\ &= \sqrt{\frac{E_r}{2B}} \int_{0}^{T_I} u(t)\bar{b}(t)e^{-i2\pi ft}\, dt\Big|_{f=0} \\ &= \sqrt{\frac{E_r}{2B}}\, U(f) * \bar{B}(f)\big|_{f=0} \\ &= \sqrt{\frac{E_r}{2B}}\, U(f) * \bar{B}(f)\big|_{f=0}, \end{aligned} \tag{42}$$

with $U(f)$ and $\bar{B}(f)$ being the Fourier transforms of $u(t)$ and $b(t)$, respectively. To compute $U(f)$ we write $u(t)$ in the form

$$u(t) = \sqrt{2P_u}\, [\cos(\varphi_0)\cos(2\pi f_I t) - \sin(\varphi_0)\sin(2\pi f_I t)];$$

its Fourier transform is

$$U(f) = \sqrt{2P_u}\, \left\{\frac{1}{2}[\delta(f+f_I) + \delta(f-f_I)]\cos(\varphi_0) - \frac{i}{2}[\delta(f+f_I) - \delta(f-f_I)]\sin(\varphi_0)\right\}. \tag{43}$$

To simplify the derivation we substitute $\bar{B}(f)$ by it's equivalent lowpass transform

$$\bar{B}(f) = \frac{1}{2}[\bar{B}_l(f - f_I) + \bar{B}_l^*(-(f + f_I))], \tag{44}$$

see the definition of the inverse complex baseband transform in spectral representation. Note that for convenience, the frequency shift in the equivalent baseband transform is set to the frequency f_I of the narrowband interference signal $u(t)$. With (43) and (44) we can express (42) as

$$\begin{aligned} \rho_{u,b} &= \sqrt{\frac{E_r}{2B}}\, U(f) * \bar{B}(f)\big|_{f=0} \\ &= \sqrt{\frac{P_u E_r}{4B}} \left\{ \frac{1}{2}[\bar{B}_l(f - 2f_I) + \bar{B}_l^*(-f) + \bar{B}_l(f) + \bar{B}_l^*(f - 2f_I)] \cos(\varphi_0) \right. \\ &\left. - \frac{i}{2}[\bar{B}_l(f) + \bar{B}_l^*(-f - 2f_I) - \bar{B}_l(f - 2f_I) - \bar{B}_l^*(-f)] \sin(\varphi_0) \right\}_{f=0}. \end{aligned} \tag{45}$$

The terms $\bar{B}_l(f - 2f_I)$ and $\bar{B}_l(-f - 2f_I)$ are zero for $f = 0$ and for any choice of f_I from the signal frequency band $[f_0 - B/2, f_0 + B/2]$. Thus,

$$\rho_{u,b} = \sqrt{\frac{P_u E_r}{4B}} \left\{ \Re\left\{\bar{B}_l(0)\right\} \cos(\varphi_0) + \Im\left\{\bar{B}_l(0)\right\} \sin(\varphi_0) \right\}. \tag{46}$$

From this we conclude that, $\rho_{u,b}$ is a weighted sum of two statistically independent Gaussian random variables and thus, Gaussian distributed with variance

$$\begin{aligned} \sigma_\rho^2 &= \mathbb{E}\left\{\rho_{u,b}^2\right\} \\ &= \frac{P_u E_r}{4B} \mathbb{E}\left\{ \bar{B}_{r,l}^2(0)\cos^2(\varphi_0) + 2\bar{B}_{r,l}(0)\bar{B}_{i,l}(0)\cos(\varphi_0)\sin(\varphi_0) + \bar{B}_{i,l}^2(0)\sin^2(\varphi_0) \right\} \\ &= \frac{P_u E_r}{4B} \left\{ \mathbb{E}\left\{\bar{B}_{r,l}^2(0)\right\}\cos^2(\varphi_0) + \mathbb{E}\left\{\bar{B}_{i,l}^2(0)\right\}\sin^2(\varphi_0) \right\} \\ &= \frac{P_u E_r}{4B} \left\{ 2\eta(T_I)\cos^2(\varphi_0) + 2\eta(T_I)\sin^2(\varphi_0) \right\} \\ &= \frac{P_u\, E_r \eta(T_I)}{2B}, \end{aligned} \tag{47}$$

where we used the statistical independence of the random variables $\bar{B}_{r,l}$ and $\bar{B}_{i,l}$ and together with 41). With (47) and since $\rho_{u,b}$ is Gaussian distributed, $\rho_{u,b}$ is characterized by

$$\rho_{u,b} \sim \mathcal{N}\left(0, \frac{P_u\, E_r\, \eta(T_I)}{2B}\right). \tag{48}$$

This result is confirmed by simulation. Note that the realization of $\rho_{u,b}$ is a deterministic function of the current channel realization $b(t)$ and the phase φ_0 of the interference signal $u(t)$. The factor one half is explained because only half of the power spectral density $E_r B(f)/B$ is captured in the integral (37), as the other half is orthogonal to the interference signal $u(t)$.

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