RZ 3638 (# 99648) 08/29/05 Electrical Engineering 4 pages

Research Report

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Simple Ergodic and Outage Capacity Expressions for Correlated Diversity Ricean Fading Channels

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Abstract—Multiple-antenna systems have been shown to achieve very high spectral efficiencies. In this paper, we derive simple single-integral expressions for the ergodic and outage capacity of a diversity system in correlated Ricean fading channels, where the channel coefficients are assumed to be known to the receiver only. For illustration purpose, we present numerical results showing the effect of channel correlation, Ricean components, angular spread and multipath components in an orthogonal frequency-division multiplexing (OFDM) system.

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Index Terms—Ergodic capacity, outage capacity, correlated fading channels, wireless SIMO systems, expected logarithm of hermitian quadratic form in complex Gaussian random variables.

I. INTRODUCTION

MULTIPLE-antenna concepts have received considerable attention in the recent history of wireless communication systems. Significant increases in spectral efficiency can by achieved by exploiting the randomness in multipath propagation. Relevant literature, however, is limited to very specific fading channel characteristics to keep the analysis tractable. A common assumption is to model the propagation coefficients between transmit and receive antennas as identically and independently distributed (i.i.d) zero-mean complex Gaussian random variables (CGRV) [1], which corresponds to a rich scattering environment. In practical scenarios, however, the coefficients are correlated and have nonzero mean because of limited scattering, insufficient antenna spacing or line-ofsight components.

In this paper, we investigate a diversity system in correlated Ricean fading channels, where the channel coefficients are assumed to be known to the receiver only. Related work is either restricted to Rayleigh fading channels [1], [2], i.i.d Ricean channels [3] [4, App. J] or uses chi-square [5] or asymptotic approximations to compute the ergodic capacity. The contributions of this paper are an exact single-integral expression for the ergodic capacity and, for non-ergodic channels, a single-integral expression for the outage capacity.

The remainder of the paper is organized as follows. The system model is introduced in Sect. II. Single-integral expressions for the ergodic and outage capacity are subsequently derived in Sect. III and Sect. IV, respectively. Numerical examples highlighting the effect of channel correlation, Ricean components, angular spread and multipath components in an orthogonal frequency-division multiplexing (OFDM) system are presented in Sect. V, and conclusions are drawn in Sect. VI.

II. SYSTEM MODEL

We consider a single-input multiple-output (SIMO) diversity system with N receive antennas. The channel is modeled as a multivariate circularly symmetric CGRV

$$\mathbf{h} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{\mu}_{\mathbf{h}}, \boldsymbol{\Sigma}_{\mathbf{h}\mathbf{h}}),$$
 (1)

with mean vector $\boldsymbol{\mu}_{\mathbf{h}} = \mathrm{E} \{\mathbf{h}\}\$ and covariance matrix $\boldsymbol{\Sigma}_{\mathbf{h}\mathbf{h}} = \mathrm{E} \{(\mathbf{h} - \boldsymbol{\mu}_{\mathbf{h}})(\mathbf{h} - \boldsymbol{\mu}_{\mathbf{h}})^{\mathrm{H}}\}\$. By defining the transmitted signal x and an additive complex Gaussian noise vector $\mathbf{n} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{\Sigma}_{\mathbf{n}\mathbf{n}})$, the system can be described as

$$\mathbf{y} = x\mathbf{h} + \mathbf{n},\tag{2}$$

where y represents the vector of observations at the N receive antennas. We assume Σ_{nn} to be non-singular and furthermore perfect channel-state information (CSI) at the receiver, but no CSI at the transmitter.

Conditioned on **h**, the maximum mutual information between x and y is achieved using a complex Gaussian input distribution $x \sim \mathcal{N}_{\mathbb{C}}(0, \mathcal{E}_s)$ and is known to be [1]

$$I(\mathbf{X}; \mathbf{Y}|\mathbf{h}) = \log(\mathbf{h}^{\mathrm{H}}\mathbf{A}\mathbf{h} + 1), \qquad (3)$$

where $\mathbf{A} = \mathcal{E}_s \Sigma_{nn}^{-1}$ is a hermitian positive definite matrix.

For the next two sections devoted to compute the ergodic capacity and the outage capacity, we put forward the following known results. The term $Q = \mathbf{h}^{\mathrm{H}} \mathbf{A} \mathbf{h}$ represents a hermitian quadratic form in CGRV with probability density function (PDF) $f_Q(q)$. Although $f_Q(q)$ can only be expressed as infinite series [6], its (two-sided) Laplace transform can be written in compact form as [7]

$$\Phi_Q(s) = \mathbf{E}_{\mathbf{h}} \left\{ \exp\left(-s\mathbf{h}^{\mathbf{H}}\mathbf{A}\mathbf{h}\right) \right\}$$

$$= \frac{\exp\left(-s\boldsymbol{\mu}_{\mathbf{h}}^{\mathbf{H}}\mathbf{A}(\mathbf{I} + s\boldsymbol{\Sigma}_{\mathbf{h}\mathbf{h}}\mathbf{A})^{-1}\boldsymbol{\mu}_{\mathbf{h}}\right)}{\det\left[\mathbf{I} + s\boldsymbol{\Sigma}_{\mathbf{h}\mathbf{h}}\mathbf{A}\right]}$$

$$= \prod_{i=1}^{N} \frac{\exp\left(\frac{-s\lambda_i|\nu_i|^2}{(1+s\lambda_i)}\right)}{(1+s\lambda_i)}, \quad (4)$$

where the last representation follows from to the eigendecomposition $\Sigma_{hh}A = V^{-1}\Delta V$ so that Δ is the diagonal matrix of non-negative eigenvalues λ_i , i = 1, ..., N, and the term $\lambda_i |\nu_i|^2$ in the exponent corresponds to the *i*-th diagonal element of the matrix $V \mu_h \mu_h^H A V^{-1}$.

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III. ERGODIC CAPACITY

If the transmission time is long enough to reveal the longterm ergodic properties of the channel, a capacity in the Shannon sense exists and is given by the *ergodic capacity* defined as [1]

$$C_{\rm E} = {\rm E}_{\mathbf{h}} \left\{ I\left(\mathbf{X}; \mathbf{Y} | \mathbf{h}\right) \right\}.$$
(5)

The non-linear log function complicates the evaluation of the expectation. The characteristic function of $I(\mathbf{X}; \mathbf{Y}|\mathbf{h})$ in (3) is given by

$$\Psi_{I}(j\xi) = \mathcal{E}_{I}[\exp\left(j\xi I\right)] = \mathcal{E}_{\mathbf{h}}\left[\left(\mathbf{h}^{\mathrm{H}}\mathbf{A}\mathbf{h}+1\right)^{j\xi}\right].$$
 (6)

The ergodic capacity (5) can then be evaluated by

$$C_{\rm E} = -j \frac{\mathrm{d}}{\mathrm{d}\xi} \Psi_I(j\xi) \Big|_{\xi=0}$$

=
$$\lim_{\epsilon \to +0} -j \frac{\mathrm{d}}{\mathrm{d}\xi} \mathbf{E}_{\mathbf{h}} \left\{ \left(\mathbf{h}^{\rm H} \mathbf{A} \mathbf{h} + 1 \right)^{-(\epsilon - j\xi)} \right\} \Big|_{\xi=0}, \quad (7)$$

where we introduced an $\epsilon \in \Re^+$, which allows us to rewrite (7) using the gamma integral

$$q^{-t} = \frac{1}{\Gamma(t)} \int_0^\infty z^{t-1} \exp(-zq) \,\mathrm{d}z, \quad \operatorname{Re}(t) > 0 \quad (8)$$

and the substitutions $t = \epsilon - j\xi$ and $q = \mathbf{h}^{\mathrm{H}}\mathbf{A}\mathbf{h} + 1$ as

$$C_{\rm E} = \lim_{\epsilon \to +0} \int_0^\infty -j \frac{\mathrm{d}}{\mathrm{d}\xi} \frac{z^{\epsilon-j\xi-1}}{\Gamma(\epsilon-j\xi)} \Big|_{\xi=0} \frac{\mathrm{E}_{\mathbf{h}} \{\exp\left(-z\mathbf{h}^{\rm H}\mathbf{A}\mathbf{h}\right)\}}{\exp\left(z\right)} \,\mathrm{d}z$$
$$= \lim_{\epsilon \to +0} \int_0^\infty \frac{\psi(\epsilon) - \log\left(z\right)}{\Gamma(\epsilon) \, z^{1-\epsilon}} \frac{\mathrm{E}_{\mathbf{h}} \{\exp\left(-z\mathbf{h}^{\rm H}\mathbf{A}\mathbf{h}\right)\}}{\exp\left(z\right)} \,\mathrm{d}z. \tag{9}$$

Here $\Gamma(.)$ and $\psi(.)$ represent Euler's gamma and psi function [8, Eqs. (8.310), (8.360)], respectively. The term $E_{\mathbf{h}} \{ \exp(-z\mathbf{h}^{\mathrm{H}}\mathbf{A}\mathbf{h}) \}$ can be interpreted as the Laplace transform of the PDF $f_Q(q)$ shown in (4), if z is replaced by $s = \alpha + jw$.

Direct evaluation of (9) is not feasible owing to the singularities on the real axes. Instead, we expand z to the complex domain s and resort to a contour integration

$$K_{\Omega} = \int_{\Omega} \frac{\psi(\epsilon) - \log(s)}{\Gamma(\epsilon) s^{1-\epsilon}} \frac{\Phi_Q(s)}{\exp(s)} \,\mathrm{d}s \tag{10}$$

along the path $\Omega = \sum_{i=1}^{6} \Omega_i$ as shown in Fig. 1. If c is chosen such that $0 < c < \min_i 1/\lambda_i$, the integrand in (10) is analytic within Ω and we conclude that

$$K_{\Omega} = \sum_{i=1}^{6} K_{\Omega_i} = 0, \tag{11}$$

where K_{Ω_i} represents the integral along the path Ω_i . Our goal is to find an expression for

$$C_{\rm E} = \lim_{\epsilon \to +0} \lim_{\substack{R \to \infty \\ \delta \to 0}} K_{\Omega_1}.$$
 (12)

Note that we define the complex logarithm $\log(s)$ such that its singular branch lies on the positive real axis, i.e. for $s = |s| \exp(j\phi)$, we define

$$\log\left(s\right) = \log\left|s\right| + j\phi, \quad 0 \le \phi < 2\pi.$$
(13)



Fig. 1. Contour integral in s-plane.

Assessment of the integration in (10) along Ω_2 , Ω_4 and Ω_6 using an upper bound argument shows that for any finite c, we have $\lim_{R\to\infty} K_{\Omega_2} = 0$, $\lim_{R\to\infty} K_{\Omega_4} = 0$ and $\lim_{\delta\to 0} K_{\Omega_6} = 0$, so that (11) simplifies to

$$\lim_{\substack{R \to \infty \\ \delta \to 0}} K_{\Omega_1} + K_{\Omega_3} + K_{\Omega_5} = 0.$$
(14)

Furthermore, owing to (10) and (13), K_{Ω_5} is related to K_{Ω_1} by

$$K_{\Omega_5} = -\exp\left(j2\pi\epsilon\right) \left[K_{\Omega_1} - 2\pi j\tilde{K}_{\Omega_1}\right],\tag{15}$$

where \tilde{K}_{Ω_i} represents the integral

$$\tilde{K}_{\Omega_i} = \int_{\Omega_i} \frac{1}{\Gamma(\epsilon) \ s^{1-\epsilon}} \frac{\Phi_Q(s)}{\exp\left(s\right)} \,\mathrm{d}s. \tag{16}$$

By substitution of K_{Ω_5} in (14) by (15) and solving for K_{Ω_1} , we obtain

$$\lim_{\substack{R\to\infty\\\delta\to0}} K_{\Omega_1} = \lim_{\substack{R\to\infty\\\delta\to0}} \frac{-1}{1-\exp\left(j2\pi\epsilon\right)} \left[K_{\Omega_3} + \frac{2\pi j \tilde{K}_{\Omega_1}}{\exp(-j2\pi\epsilon)} \right].$$
(17)

By an auxiliary calculation, we redo the analysis steps (11) - (14) shown above for the contour integral (16) to further show that

$$\tilde{K}_{\Omega_1} = \frac{-1}{1 - \exp\left(j2\pi\epsilon\right)} \tilde{K}_{\Omega_3}.$$
(18)

In the limit as $R \to \infty$ and $\delta \to 0$, by combining (18) and (17) and the definition of our contour integrals (10) and (16), we get the expression

$$\lim_{\substack{R \to \infty \\ \delta \to 0}} K_{\Omega_1} = \int_{-c-j\infty}^{-c+j\infty} \frac{\psi(\epsilon) - \log\left(s\right) - \frac{2\pi j \exp\left(j2\pi\epsilon\right)}{1 - \exp\left(j2\pi\epsilon\right)}}{\Gamma(\epsilon) \, s^{1-\epsilon} \left[1 - \exp\left(j2\pi\epsilon\right)\right]} \frac{\Phi_Q(s)}{\exp\left(s\right)} \, \mathrm{d}s.$$
(19)

The remaining limit $\epsilon \to +0$ exists, and we finally obtain the desired single-integral expression for the ergodic capacity (12)

$$C_{\rm E} = \frac{1}{2\pi j} \int_{-c-j\infty}^{-c+j\infty} \frac{(\gamma - j\pi + \log(s))}{s} \frac{\Phi_Q(s)}{\exp(s)} \,\mathrm{d}s \qquad (20)$$

with region of convergence (ROC) $0 < c < \min_i 1/\lambda_i$ and $\gamma \approx 0.577216$ denoting Euler's constant.

IV. OUTAGE CAPACITY

If the transmission time is not long enough to reveal the long-term ergodic properties of the fading channel, the concept of *outage capacity* is evoked, in which the capacity is viewed as a random variable. The outage capacity $C_{\rm O}$ is the capacity guaranteed for (100 - p)% of the channel realizations

$$\Pr\left[C \le C_{\rm O}\right] = p\,\% \tag{21}$$

and can be achieved by assuming coding within one channel realization, but the length of this block goes to infinity (noise averaging).

Next we derive a single-integral expression for the cumulative distribution function (CDF) (21). With

$$\Pr[C \leq C_{\rm O}] = \Pr[I(\mathbf{X}; \mathbf{Y} | \mathbf{h}) \leq \log(\exp(C_{\rm O}))]$$
$$= \Pr[\mathbf{h}^{\rm H} \mathbf{A} \mathbf{h} + 1 - \exp(C_{\rm O}) \leq 0], \quad (22)$$

we can again solve the problem in the Laplace domain using (4) and by interchanging inverse Laplace transform and PDF integration obtain

$$\Pr\left[C \le C_{\rm O}\right] = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} \frac{\exp(s\exp(C_{\rm O}))}{s} \frac{\Phi_Q(s)}{\exp(s)} \,\mathrm{d}s, \quad (23)$$

where the ROC is given by 0 < c. Alternatively, the evaluation of $\Pr[C > C_O] = 1 - \Pr[C \le C_O]$ leads to

$$\Pr\left[C > C_{\rm O}\right] = \frac{1}{2\pi j} \int_{-c-j\infty}^{-c+j\infty} \frac{\exp(s\exp(C_{\rm O}))}{s} \frac{\Phi_Q(s)}{\exp(s)} \,\mathrm{d}s, \tag{24}$$

with ROC $0 < c < \min_i 1/\lambda_i$, which turns out to achieve faster convergence in the remaining numerical integration. The relation to (23) is determined by shifting the integration strip to the left half-plane of *s* while correcting for the residue of the simple pole at zero.

The mean capacity (ergodic capacity) can be derived from the CDF above using partial integration

$$C_{\rm E} = \int_0^\infty C_{\rm O} \left(\frac{\mathrm{d}}{\mathrm{d}C_{\rm O}} \Pr\left[C \le C_{\rm O}\right] \right) \,\mathrm{d}C_{\rm O}$$
$$= C_{\rm O} \Pr\left[C \le C_{\rm O}\right] \Big|_0^\infty - \int_0^\infty \Pr\left[C \le C_{\rm O}\right] \,\mathrm{d}C_{\rm O}, (25)$$

which, owing to the properties of densities, leads to

$$C_{\rm E} = \int_0^\infty \left(1 - \Pr\left[C \le C_{\rm O}\right] \right) \, \mathrm{d}C_{\rm O}.$$
 (26)

Applying the identity [8, Eq. 3.327]

$$-\operatorname{Ei}(-s) = \int_{1}^{\infty} \frac{\exp\left(st\right)}{t} dt$$
$$= \int_{0}^{\infty} \exp\left(s\exp\left(C_{O}\right)\right) dC_{O}, \qquad \operatorname{Re}\{s\} < 0 \ (27)$$

and inserting (24) in (26), we finally get

$$C_{\rm E} = \frac{1}{2\pi j} \int_{-c-j\infty}^{-c+j\infty} \frac{-{\rm Ei}(-s)}{s} \frac{\Phi_Q(s)}{\exp\left(s\right)} \,\mathrm{d}s \tag{28}$$

with ROC defined by $0 < c < \min_i 1/\lambda_i$. Note that (28) has a similar structure as (20), but a different and more complex weighting function.

V. APPLICATION AND NUMERICAL RESULTS

The remaining integration in the ergodic capacity expression (20) and the outage capacity expression (23) can easily be evaluated using numerical methods [9]. In the following, we use the Gauss–Chebyshev quadrature [10] to evaluate these expressions and Monte Carlo simulations for verification. We consider a SIMO system employing OFDM over a broadband channel that follows the model introduced in [11], [12].

We assume the channel impulse response to consist of L equally spaced taps resulting from different uncorrelated clusters. Thus, the channel's frequency response is given by

$$\mathbf{h}(\theta) = \sum_{l=0}^{L-1} \mathbf{h}_l \exp\left(-j2\pi\theta l\right), \quad 0 \le \theta < 1, \quad (29)$$

where the *l*-th tap \mathbf{h}_l is an $N \times 1$ vector representing the response of the receive antenna array to the impinging wave. Furthermore, each tap can be decomposed into direct and specular components according to

$$\mathbf{h}_{l} = \sqrt{\sigma_{l}^{2}} \left(\sqrt{\frac{\kappa_{l}}{1 + \kappa_{l}}} \bar{\mathbf{h}}_{l} + \sqrt{\frac{1}{1 + \kappa_{l}}} \boldsymbol{\Sigma}_{l}^{1/2} \tilde{\mathbf{h}}_{l} \right), \qquad (30)$$

where σ_l^2 , κ_l and $\Sigma_l^{1/2}$ denote the power delay profile, the Ricean *K*-factor and the receive spatial correlation matrix for tap *l*, respectively. The direct component vector $\bar{\mathbf{h}}_l$ is fixed and has unit energy entries, whereas $\tilde{\mathbf{h}}_l$ is a zero mean complex Gaussian random vector $\tilde{\mathbf{h}}_l \sim \mathcal{N}_{\mathbb{C}}(0, \mathbf{I})$. As the taps are assumed Gaussian, the channel's frequency response at a given frequency θ is a CGRV. Assuming that only the first tap has a line-of-sight component, the distribution is completely determined by

$$\mathbf{h}(\theta) \sim \mathcal{N}_{\mathbb{C}} \left(\sqrt{\sigma_0^2} \sqrt{\frac{\kappa_0}{1+\kappa_0}} \bar{\mathbf{h}}_0, \sum_{l=0}^{L-1} \sigma_l^2 \frac{1}{1+\kappa_l} \boldsymbol{\Sigma}_l \right) \quad (31)$$

and is wide-sense stationary in the frequency domain. We assume a uniform linear antenna array at the receiver and use the correlation model considered in [11] to determine the matrices Σ_l . For small cluster angle spreads, the correlation function between receive antennas m and m' can be approximated as [13]

$$\rho(\delta n, \bar{\theta}, \sigma_{\theta}) \approx \exp\left(-j2\pi n\delta\cos\bar{\theta} - \frac{1}{2}(2\pi n\delta\sigma_{\theta}\sin\bar{\theta})^{2}\right),$$

where n = m - m', δ is the antenna spacing in wavelengths, $\bar{\theta}$ is the mean angle of arrival and σ_{θ} denotes the angular spread. Thus, the entries of the spatial correlation matrix are given by

$$[\mathbf{\Sigma}_l]_{m',m} = \rho(\delta(m - m'), \theta_l, \sigma_{\theta,l}).$$
(32)

We use the single-integral expressions derived above to examine the impact of some parameters in the channel model on information rates. For an analysis of the influence of the propagation environment on the capacity of general MIMO systems, the reader is referred to [12]. In what follows, we assume uncorrelated noise $\sum_{nn} = \sigma_n^2 \mathbf{I}$ and normalize the energy of the channel $\sum_{l=0}^{L-1} \sigma_l^2 = 1$ so that the SNR can be defined as $\rho = \mathcal{E}_s / \sigma_n^2$.

First, we consider a flat-fading environment, where the channel consists of a single purely specular tap. The mean



Fig. 2. 10%-outage capacities for different numbers of receive antennas, $N \in \{2, 4\}$, and angular spreads, $\sigma_{\theta}^2 \in \{0.01, 0.05, 0.09\}$.



Fig. 3. Impact of the angular spread σ_{θ} on ergodic and 10%-outage capacities. The receiver has N=4 antennas.

angle of arrival is fixed at $\bar{\theta} = \pi/2$, antenna spacing $\delta = 1/2$, and different values of angular spread σ_{θ} are considered. The resulting 10% outage capacities are plotted versus SNR in Fig. 2 for N = 2 and N = 4 receive antennas. Clearly, the angular spread plays an important role from the perspective of mutual information: a 4-antenna system can be outperformed by a 2-antenna one that experiences a larger angular spread.

The dependencies of ergodic and outage capacities on the angular spread are compared with each other in Fig. 3. Obviously, angular spread is beneficial for both. However, the increase is more pronounced in the outage case, where higher rates can be guaranteed at a given outage level as the angular spread increases.

In the third example, a frequency-selective channel comprising L equal-energy multipath components is considered. The mean angle of arrival for every tap is uniformly distributed in the interval $[0, 2\pi)$, and the angular spread for each cluster is set to zero. As it can be observed in Fig. 4, the tail probability of the mutual information at $\rho = 10$ dB decreases for increasing number of taps L. A direct component ($\kappa_0 = 1$) stabilizes the link and higher rates are supported at low outage probabilities.



Fig. 4. Tail behavior of the mutual information for different numbers of multipath taps L, Ricean K-factor κ_0 and N = 4 antennas at SNR $\rho = 10$ dB.

VI. CONCLUSIONS

In this paper, simple single-integral expressions have been derived for the ergodic and outage capacities of a diversity system in correlated Ricean fading channels, where the channel coefficients are assumed to be known to the receiver only. The capacity expression have been used in numerical examples to evaluate the effect of channel correlation, Ricean components, angular spread and multipath components in an OFDM-based receive diversity system.

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