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Spatial Multiplexing in the Single-Relay MIMO Channel

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Abstract— **This work considers the problem of relaying spatially multiplexed signals between a source and a destination using a non-regenerative relay equipped with multiple antennas. In addition to the usual power amplification, the relay is assumed to process the received data vector before forwarding it to the destination. We present two linear filtering techniques that perform this processing in an optimal manner when channel state information (CSI) on the backward and forward channels is available at the relay. Firstly, we derive the relay filter that minimizes the pairwise error probability (PEP) assuming maximum-likelihood (ML) detection at the destination. Secondly, we simultaneously optimize the relay and destination filters in the mean-square error (MSE) sense. The proposed relaying techniques are shown to achieve error performance improvements, especially when the number of antennas at the relay is increased.**

I. Introduction

The use of multiple antennas in wireless relaying networks offers significant gains in terms of spectral efficiency and link reliability [1]. These advantages convert MIMO relaying into a key technology for future high data-rate wireless communication systems in cellular and ad-hoc networks. Both non-regenerative and regenerative relays can exploit the benefits stemming from the use of multiple antennas. However, regenerative relaying incurs a considerable implementation complexity that is not always desirable or affordable. On the other hand, simpler non-regenerative techniques (i.e. where information data is copied, processed and retransmitted, but not regenerated) usually take a performance hit in comparison with regenerative ones. As a consequence, it is desirable to investigate how and to what extent relay processing can enhance their performance.

Wireless relaying has been shown to achieve significant gains [2]–[4]. In particular, the use of relay nodes introduces an additional source of diversity that can be exploited by an appropriate signal design. The work in [1] studies the capacity scaling laws in large MIMO relay networks, and quantifies the benefits of using channel state information (CSI) at the relays from a network capacity standpoint. Results on optimal relaying when multiple source-destination pairs communicate simultaneously can be found in [5]. In the absence of relays, the optimal transmitter/receiver filter design for multiplexed signals has been considered in [6]–[8], where the latter reference provides a comprehensive study of the MIMO channel case. A generic capacity analysis of the MIMO relay channel is reported in [9], and [10] derives the linear relay processing that achieves maximum instantaneous capacity.

In this work, we study the problem of relaying spatially-multiplexed signals between a source node and a destination node using a single relay equipped with multiple antennas. The end-to-end signal transmission is assumed to span two time slots of a TDMA-based system or, equivalently, two orthogonal frequency bands of an FDMA-based system. The relay amplifies and linearly processes the signal received from the source before forwarding it to the destination. Using perfect CSI on the backward (i.e. source to relay) and forward (i.e. relay to destination) channels, we present two non-regenerative processing methods that improve the error performance and that are optimal in the maximum-likelihood (ML) and mean-square error (MSE) senses. We illustrate the performance gains with respect to conventional non-regenerative relaying by simulations means and demonstrate that our optimized processing techniques are particularly useful when the relay has more antennas than the other nodes.

A. Notation

The superscript ^{H} stands for conjugate transpose, $\mathcal{E}\left\{.\right\}$ denotes the expectation operator and $\mathrm{Tr}\left(\mathbf{A}\right)$ stands for the trace of matrix $\mathbf{A}$. The set of all $m \times n$ matrices over the complex field is denoted by $\mathcal{M}_{m,n}$. The random vector $\mathbf{x} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{R}\right)$ is a zero-mean circularly symmetric complex Gaussian random vector with covariance matrix $\mathbf{R}$. The Frobenius norm of matrix $\mathbf{A}$ is denoted as $||\mathbf{A}||_{\mathrm{F}} = \mathrm{Tr}\left(\mathbf{A}^H\mathbf{A}\right)^{1/2}$.

II. System Model and Assumptions

We consider a relaying network composed of a source (S), a relay (R) and a destination (D) that have $\mathrm{M_s}$, $\mathrm{M_r}$ and $\mathrm{M_d}$ antennas respectively. The transmitter operates in the spatial-multiplexing mode, i.e. $\mathrm{M_d} \geq \mathrm{M_s}$, and we assume $\mathrm{M_r} \geq \mathrm{M_s}$ to avoid relay-induced signal-space collisions. We focus on a transmission strategy where no direct communication path exists between S and D (e.g. they might be too far apart) and R is constrained to linear processing and power amplification.

The wireless channels are described by a flat-fading matrix that does not change over the packet duration. We denote the backward channel (S $\rightarrow$ R) by $\mathbf{H} \in \mathcal{M}_{\mathrm{M_r},\mathrm{M_s}}$, and the forward channel (R $\rightarrow$ D) by $\mathbf{G} \in \mathcal{M}_{\mathrm{M_d},\mathrm{M_r}}$. Both R and D (but not S) have access to the current channel realizations, and we assume that CSI updates occur as dictated by the channel variation

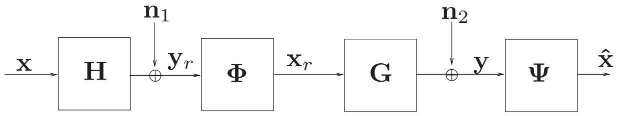

Fig. 1. MIMO relay channel, where $\mathbf{H}$ denotes the backward channel, $\mathbf{G}$ denotes the forward channel, and $\mathbf{\Phi}$ and $\mathbf{\Psi}$ denote the processing performed at the relay and at the destination respectively.

rate. Let us define the singular value decompositions (SVD) for both channel matrices as follows:

$$\mathbf{H} = \mathbf{T}\mathbf{\Sigma}\mathbf{U}^H,\ \mathbf{T}\in\mathcal{M}_{\mathrm{M_r}},\ \mathbf{U}\in\mathcal{M}_{\mathrm{M_s}} \quad (1)$$
$$\mathbf{G} = \mathbf{V}\mathbf{\Lambda}\mathbf{W}^H,\ \mathbf{V}\in\mathcal{M}_{\mathrm{M_d}},\ \mathbf{W}\in\mathcal{M}_{\mathrm{M_r}} \quad (2)$$

where the matrix $\mathbf{\Sigma} = [\sigma_{k,j}] \in \mathcal{M}_{\mathrm{M_r},\mathrm{M_s}}$ has $\sigma_{k,j} = 0$, for all $k \neq j$, and $\sigma_{k,k} \equiv \sigma_k$, $k = 1, \ldots, \mathrm{M_s}$ are the singular values of $\mathbf{H}$ arranged in decreasing order. Similarly, the matrix $\mathbf{\Lambda} = [\lambda_{k,j}] \in \mathcal{M}_{\mathrm{M_d},\mathrm{M_r}}$ has $\lambda_{k,j} = 0$, for all $k \neq j$, and $\lambda_{k,k} \equiv \lambda_{\pi(k)}$, $k = 1, \ldots, \min(\mathrm{M_r}, \mathrm{M_d})$, is a permutation of the singular values of $\mathbf{G}$ arranged in decreasing order.

Node S transmits a data vector $\mathbf{x} \in \mathcal{M}_{\mathrm{M_s},1}$ whose elements are taken from a constellation with normalized energy. The signals received at R and D are given respectively by

$$\mathbf{y}_r = \sqrt{\frac{\rho_1}{\mathrm{M_s}}}\mathbf{H}\mathbf{x} + \mathbf{n}_1 \ \text{ and } \ \mathbf{y} = \sqrt{\frac{\rho_2}{\mathrm{M_r}}}\mathbf{G}\mathbf{x}_r + \mathbf{n}_2, \quad (3)$$

where the SNRs ρ_1 and ρ_2 include the path-loss, and the noise vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ have i.i.d. entries distributed according to $\mathcal{CN}(0,1)$. The signal transmitted by the relay is obtained from the incoming signal as $\mathbf{x}_r = \sqrt{s}\mathbf{\Phi}\mathbf{y}_r$, where s is the power amplification factor and $\mathbf{\Phi}$ denotes a linear operation constrained in power by $\mathrm{Tr}\left(\mathbf{\Phi}\mathbf{\Phi}^H\right) \leq \mathrm{M_r}$. The factor s compensates for the signal attenuation caused by the propagation from S to R. More precisely, the amplification ensures that $s\ \mathrm{Tr}\left(\mathbf{y}_r\mathbf{y}_r^H\right) = \mathrm{M_r}$, which yields $s = \frac{1}{\rho_1+1}$ on average. The end-to-end signal model can be written as

$$\mathbf{y} = \sqrt{\gamma}\ \mathbf{G}\mathbf{\Phi}\mathbf{H}\mathbf{x} + \mathbf{n}, \quad (4)$$

where we have introduced $\gamma = \frac{\rho_1\rho_2}{\mathrm{M_s}\mathrm{M_r}(\rho_1+1)}$ to simplify the notation. The equivalent noise term is distributed according to $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R_n})$, where

$$\mathbf{R_n} = \alpha\mathbf{G}\mathbf{\Phi}\mathbf{\Phi}^H\mathbf{G}^H + \mathbf{I}_{\mathrm{M_d}}, \quad (5)$$

and we have introduced $\alpha = \frac{\rho_2}{\mathrm{M_r}(\rho_1+1)}$. The destination applies the receiver $\mathbf{\Psi}$ to the incoming signal $\mathbf{y}$ to produce an estimate of the transmitted signal $\hat{\mathbf{x}}$. Figure 1 depicts a block diagram of the overall input-output relation.

III. Maximum Likelihood Optimization

Consider the end-to-end signal model in (4). Conditioned on the transmit vector $\mathbf{x}$ and on the product $\mathbf{G}\mathbf{\Phi}\mathbf{H}$, the received signal is distributed according to $\mathcal{CN}\left(\sqrt{\gamma}\mathbf{G}\mathbf{\Phi}\mathbf{H}\mathbf{x}, \mathbf{R_n}\right)$, where $\mathbf{R_n}$ is defined in (5). Consequently, the ML receiver computes

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \left\| \mathbf{R_n}^{-\frac{1}{2}}(\mathbf{y} - \sqrt{\gamma}\mathbf{G}\mathbf{\Phi}\mathbf{H}\mathbf{x}) \right\|_{\mathrm{F}}^2. \quad (6)$$

This nonlinear processing is implemented by the operation $\mathbf{\Psi}$ at the destination (see Fig. 1). The ML pairwise error probability (PEP), that is, the probability that the ML receiver erroneously decides in favor of vector $\mathbf{s}$ when $\mathbf{x}$ was actually transmitted, is given by

$$\Pr(\mathbf{x} \to \mathbf{s}|\mathbf{G}\mathbf{\Phi}\mathbf{H}) = \mathcal{Q}\left(\sqrt{\frac{\gamma}{2}\left\|\mathbf{R_n}^{-\frac{1}{2}}\mathbf{G}\mathbf{\Phi}\mathbf{H}\mathbf{e}\right\|_{\mathrm{F}}^2}\right), \quad (7)$$

where $\mathcal{Q}(.)$ is the Gaussian error function and $\mathbf{e} = \mathbf{x} - \mathbf{s}$ denotes the codeword difference vector. The relay filter that minimizes the PEP is given by

$$\mathbf{\Phi}^\star = \arg\max_{\mathbf{\Phi}:\|\mathbf{\Phi}\|_{\mathrm{F}}^2 \leq \mathrm{M_r}} \xi(\mathbf{\Phi}), \quad (8)$$

where $\xi(\mathbf{\Phi}) := \gamma\mathbf{e}^H\mathbf{H}^H\mathbf{\Phi}^H\mathbf{G}^H\mathbf{R_n}^{-1}\mathbf{G}\mathbf{\Phi}\mathbf{H}\mathbf{e}$. It is interesting to note that for $\alpha \gg 1$, i.e. the noise on the second hop is negligible, the cost function becomes

$$\xi(\mathbf{\Phi}) \approx \frac{\gamma}{\alpha}\mathbf{e}^H\mathbf{H}^H\mathbf{\Phi}^H\mathbf{G}^H\left(\mathbf{G}\mathbf{\Phi}\mathbf{\Phi}^H\mathbf{G}^H\right)^{-1}\mathbf{G}\mathbf{\Phi}\mathbf{H}\mathbf{e}, \quad (9)$$

where the matrix $\mathbf{\Phi}^H\mathbf{G}^H\left(\mathbf{G}\mathbf{\Phi}\mathbf{\Phi}^H\mathbf{G}^H\right)^{-1}\mathbf{G}\mathbf{\Phi} \preceq \mathbf{I}$ is the orthogonal projection onto the range space of $\mathbf{\Phi}^H\mathbf{G}^H$. Consequently, for any $\mathbf{\Phi}$, the cost function is upper bounded as

$$\xi(\mathbf{\Phi}) \leq \xi^\star := \frac{\gamma}{\alpha}\mathbf{e}^H\mathbf{H}^H\mathbf{H}\mathbf{e}, \quad (10)$$

where equality is achieved for $\mathbf{\Phi}^\star = \mathbf{G}^{-1}\mathbf{H}^H$. For general values of α, we employ the matrix inversion lemma (MIL) [11] to write

$$\mathbf{\Phi}^H\mathbf{G}^H\mathbf{R_n}^{-1}\mathbf{G}\mathbf{\Phi} = \frac{1}{\alpha}\left(\mathbf{I} - \left(\alpha\mathbf{\Phi}^H\mathbf{G}^H\mathbf{G}\mathbf{\Phi} + \mathbf{I}\right)^{-1}\right). \quad (11)$$

Inserting this expression into the cost function yields

$$\xi(\mathbf{\Phi}) = \xi^\star - \frac{\gamma}{\alpha}\ \mathbf{e}^H\mathbf{H}^H\left(\alpha\mathbf{\Phi}^H\mathbf{G}^H\mathbf{G}\mathbf{\Phi} + \mathbf{I}\right)^{-1}\mathbf{H}\mathbf{e}, \quad (12)$$

where the second term of the RHS quantifies a penalty associated with the second hop. As $\alpha \to \infty$, the penalty vanishes and the solution is that discussed above. We shall next determine the matrix $\mathbf{\Phi}$ that minimizes this penalty.

In order to find a solution that does not depend on a specific codeword difference vector $\mathbf{e}$, we use the Rayleigh-Ritz Theorem [11] to bound the penalty term as

$$\begin{aligned}&\mathbf{e}^H\mathbf{H}^H\left(\alpha\mathbf{\Phi}^H\mathbf{G}^H\mathbf{G}\mathbf{\Phi} + \mathbf{I}\right)^{-1}\mathbf{H}\mathbf{e}\\ &\qquad \leq \lambda_{\max}\left(\mathbf{H}^H\left(\alpha\mathbf{\Phi}^H\mathbf{G}^H\mathbf{G}\mathbf{\Phi} + \mathbf{I}\right)^{-1}\mathbf{H}\right)||\mathbf{e}||^2,\end{aligned}$$

where $\lambda_{\max}(\mathbf{A})$ denotes the maximum eigenvalue of $\mathbf{A}$. We shall minimize this upper bound. Assuming without loss of generality that the relay processing matrix has the structure $\tilde{\mathbf{\Phi}} = \mathbf{W}\mathbf{\Phi}\mathbf{T}^H$, the optimization problem can be recast as

$$\mathbf{\Phi}^\star = \arg\min_{\mathbf{\Phi}:\|\mathbf{\Phi}\|_{\mathrm{F}}^2 \leq \mathrm{M_r}} \lambda_{\max}\left(\mathbf{\Sigma}\left(\alpha\mathbf{\Phi}^H\mathbf{\Lambda}^2\mathbf{\Phi} + \mathbf{I}\right)^{-1}\mathbf{\Sigma}\right). \quad (13)$$

The maximal eigenvalue is minimized when $\lambda_{\max} = \lambda_{\min}$, and consequently, the optimal relay processing has to satisfy $\mathbf{Z}\ \mathbf{\Sigma}\left(\alpha\mathbf{\Phi}^H\mathbf{\Lambda}^2\mathbf{\Phi} + \mathbf{I}\right)^{-1}\mathbf{\Sigma}\mathbf{Z}^H = \mu\mathbf{I}$, where $\mathbf{Z}$ is an arbitrary unitary matrix and μ is a scaling introduced to meet the power constraint. In other words, the optimal matrix is unitary equivalent to the identity matrix. Solving for $\mathbf{\Phi}$ in the above expression yields

$$\mathbf{\Phi}^2 = \frac{1}{\mu\alpha}\mathbf{\Lambda}^{-2}\left(\mathbf{\Sigma}^2 - \mu\mathbf{I}\right)_+, \tag{14}$$

where $(a)_+ = \max(a,0)$ is used to ensure the positive semi-definiteness of the solution. Note that the optimal $\mathbf{\Phi}$ is diagonal. The constant μ is determined by the power constraint as

$$\mu = \frac{\sum_k \frac{\sigma_k^2}{\lambda_{\pi(k)}^2}}{\alpha M + \sum_k \frac{1}{\lambda_{\pi(k)}^2}}, \tag{15}$$

where the sum is taken over the active modes. To see the behavior of the solution as α grows, let us assume that all the modes are employed. Noting that $\lim_{\alpha\to\infty}\frac{\mu}{\alpha} = 0$, the cost approaches its optimal value $\xi^\star$ according to:

$$\xi(\mathbf{\Phi}) = \xi^\star - \frac{\gamma\mu}{\alpha}||\mathbf{e}||^2. \tag{16}$$

We observe that the relay rotates the backward and forward channels in order to align their eigenmodes. All the power is directly poured on the eigenmodes of the equivalent channel as a function of the singular values of $\mathbf{H}$ and $\mathbf{G}$. Note from (14) that while the weaker singular values of $\mathbf{G}$ lead to larger power allocations, the weaker singular values of $\mathbf{H}$ determine the modes to be dropped. Finally, it should be stressed that the optimal solution depends on the eigenvalue permuation π.

IV. Mean Square Error Optimization

The objective of this section is to determine the optimal linear filters that R and D can apply to their received signals. Both filters should be jointly designed to optimize the overall MSE. Recalling the system model described in Figure 1, and the signal model in (4), the output of the destination filter $\mathbf{\Psi}$ is given by

$$\hat{\mathbf{x}} = \sqrt{\gamma}\ \mathbf{\Psi}\mathbf{G}\mathbf{\Phi}\mathbf{H}\mathbf{x} + \mathbf{\Psi}\mathbf{n}, \tag{17}$$

where $\mathbf{n}$ is additive white Gaussian noise distributed according to $\mathcal{CN}(\mathbf{0},\mathbf{R_n})$. Using the MSE criterion between $\mathbf{x}$ and its estimate $\hat{\mathbf{x}}$, the optimization problem can be stated as

$$\min_{\mathbf{\Phi},\mathbf{\Psi}:\|\mathbf{\Phi}\|_{\mathrm{F}}^2\le \mathrm{M_r}} \mathrm{Tr}\left(\mathbf{C_e}\right), \tag{18}$$

where, assuming $\mathcal{E}\left\{\mathbf{x}\mathbf{x}^H\right\} = \mathbf{I}$ and $\mathcal{E}\left\{\mathbf{x}\mathbf{n}^H\right\} = \mathbf{0}$, the error covariance matrix is given by

$$\mathbf{C_e} = \left(\sqrt{\gamma}\mathbf{\Psi}\mathbf{G}\mathbf{\Phi}\mathbf{H} - \mathbf{I}\right)\left(\sqrt{\gamma}\mathbf{\Psi}\mathbf{G}\mathbf{\Phi}\mathbf{H} - \mathbf{I}\right)^H + \alpha\mathbf{\Psi}\mathbf{G}\mathbf{\Phi}\mathbf{\Phi}^H\mathbf{G}^H\mathbf{\Psi}^H + \mathbf{\Psi}\mathbf{\Psi}^H. \tag{19}$$

Denoting by $\mu \ge 0$ the Lagrange multiplier, we form the Lagrangian:

$$\mathcal{L}(\mu,\mathbf{\Phi},\mathbf{\Psi}) = \mathrm{Tr}\left(\mathbf{C_e}\right) + \mu\left(\mathrm{Tr}\left(\mathbf{\Phi}\mathbf{\Phi}^H\right) - \mathrm{M_r}\right). \tag{20}$$

A necessary and sufficient condition for optimality of a pair $(\mathbf{\Phi},\mathbf{\Psi})$ is given by the conditions:

$$\frac{\partial}{\partial\mathbf{\Phi}}\mathcal{L}(\mu,\mathbf{\Phi},\mathbf{\Psi}) = 0, \qquad \frac{\partial}{\partial\mathbf{\Psi}}\mathcal{L}(\mu,\mathbf{\Phi},\mathbf{\Psi}) = 0, \tag{21}$$

$$\mu\left(\mathrm{Tr}\left(\mathbf{\Phi}\mathbf{\Phi}^H\right) - \mathrm{M_r}\right) = 0, \quad \mathrm{Tr}\left(\mathbf{\Phi}\mathbf{\Phi}^H\right) - \mathrm{M_r} \le 0. \tag{22}$$

Considering a matrix and its Hermitian transposed as independent variables and using the matrix derivatives $\frac{\partial\mathrm{Tr}(\mathbf{AXB})}{\partial\mathbf{X}} = \mathbf{BA}$ and $\frac{\partial\mathrm{Tr}\left(\mathbf{AX}^H\mathbf{B}\right)}{\partial\mathbf{X}} = \mathbf{0}$ [12], it can be shown that (21) yields the following relations between $\mathbf{\Phi}$ and $\mathbf{\Psi}$:

$$\left(\gamma\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}\right)\mathbf{\Phi}^H\mathbf{G}^H\mathbf{\Psi}^H\mathbf{\Psi}\mathbf{G}\mathbf{\Phi} + \mu\mathbf{\Phi}^H\mathbf{\Phi} = \sqrt{\gamma}\mathbf{H}\mathbf{\Psi}\mathbf{G}\mathbf{\Phi},$$
$$\mathbf{\Psi}\mathbf{G}\mathbf{\Phi}\left(\gamma\mathbf{H}\mathbf{H}^H + \alpha\mathbf{I}\right)\mathbf{\Phi}^H\mathbf{G}^H\mathbf{\Psi}^H + \mathbf{\Psi}\mathbf{\Psi}^H = \sqrt{\gamma}\mathbf{\Psi}\mathbf{G}\mathbf{\Phi}\mathbf{H}.$$

We shall assume hereafter that $\mathrm{M_s} = \mathrm{M_r} = \mathrm{M_d}$ to simplify our exposition, but the extension to asymmetric antenna configurations is straightforward. Using the SVD of the channel matrices defined in (1) and (2), let $\mathbf{\Phi}$ and $\mathbf{\Psi}$ have the following structure

$$\mathbf{\Phi} = \mathbf{W}\mathbf{D_\Phi}\mathbf{T}^H,\ \mathbf{D_\Phi} \in \mathcal{M}_{\mathrm{M_r}} \tag{23}$$
$$\mathbf{\Psi} = \mathbf{U}\mathbf{D_\Psi}\mathbf{V}^H,\ \mathbf{D_\Psi} \in \mathcal{M}_{\mathrm{M_s},\mathrm{M_d}}, \tag{24}$$

where $\mathbf{D_\Phi} = \mathrm{diag}\left\{d_{\Phi,1}, d_{\Phi,2},\ldots,d_{\Phi,\mathrm{M_r}}\right\}$ and $\mathbf{D_\Psi} = \mathrm{diag}\left\{d_{\Psi,1}, d_{\Psi,2},\ldots,d_{\Psi,\mathrm{M_s}}\right\}$. The equations in (21) yield

$$\mathbf{D_\Phi}^2 = \left(\gamma\mathbf{\Sigma^2\Lambda^2} + \alpha\mathbf{\Lambda^2}\right)^{-1}\left(\sqrt{\frac{\gamma}{\mu}}\mathbf{\Sigma\Lambda} - \mathbf{I}\right)_+ \tag{25}$$
$$\mathbf{D_\Psi}^2 = \mu\mathbf{D_\Phi}^2, \tag{26}$$

where $(a)_+ = \max(a,0)$. Using $\mathrm{Tr}\left(\mathbf{\Phi}\mathbf{\Phi}^H\right) = \mathrm{M_r}$ and assuming that $M \le \mathrm{rank}\left(\mathbf{GH}\right)$ subchannels are used, the Lagrange multiplier μ is the solution to

$$\sqrt{\mu} = \frac{\sum_{k=1}^M\left(\gamma\delta_k^2 + \alpha\lambda_{\pi(k)}^2\right)^{-1}\sqrt{\gamma}\delta_k}{\mathrm{M_r} + \sum_{k=1}^M\left(\gamma\delta_k^2 + \alpha\lambda_{\pi(k)}^2\right)^{-1}}, \tag{27}$$

for an arbitrary permutation π and $\delta_k = \sigma(k)\lambda(\pi(k))$, $\delta_1 \ge \delta_2 \ge \ldots \ge \delta_N$. The optimal $\mu \ge 0$ should ensure that the matrices $\mathbf{D_\Phi}$ and $\mathbf{D_\Psi}$ have non-negative entries.

As in the previous method, the relay rotates the backward and forward channels in order to align their eigenmodes. The available power is directly poured on the eigenmodes of the equivalent channel as a function of the singular values of $\mathbf{H}$ and $\mathbf{G}$. When sufficient power is available, the allocation policy yields equally strong subchannels, resulting in perfect interstream interference cancellation. Note that the optimal power allocation depends on the eigenvalue permutation π.

V. Performance Simulations

In our simulations, the source and the destination have a fixed number of antennas $\mathrm{M_s} = \mathrm{M_d} = 2$ and the relay has $\mathrm{M_r} = 2, 3$ or 4 antennas. Assuming Rayleigh flat-fading, the entries of the forward and backward channel matrices are generated according to a $\mathcal{CN}(0,1)$ distribution and the performance is averaged over multiple independent realizations. We consider spatial-multiplexing of BPSK uncoded symbols (i.e. 2

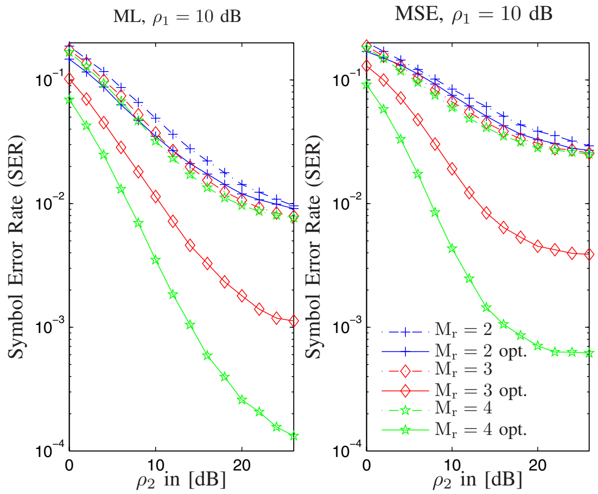

Fig. 2. Error performance comparison between ML techniques and MSE techniques at $\rho_1 = 10$ dB SNR on the backward link. Optimized relaying is depicted by solid lines, amplify-only relaying by dashed lines. The number of antennas at the relay are $\mathrm{M_r} = 2, 3$ and 4.

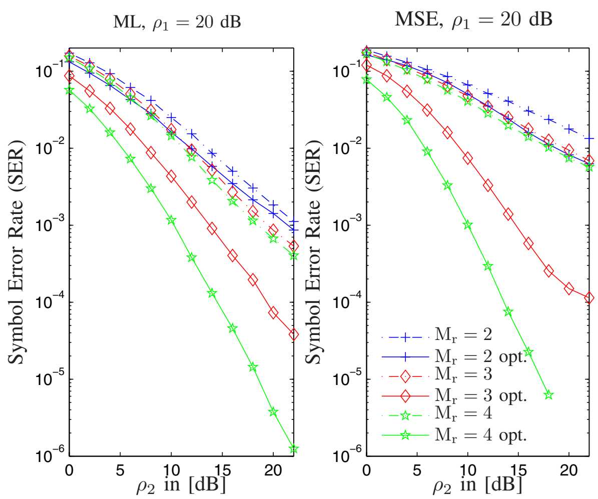

Fig. 3. Error performance comparison between ML techniques and MSE techniques at $\rho_1 = 20$ dB SNR on the backward link. Optimized relaying is depicted by solid lines, amplify-only relaying by dashed lines. The number of antennas at the relay are $\mathrm{M_r} = 2, 3$ and 4.

bits/transmission), and our optimized techniques are compared to a simple power amplification (the signal power is scaled at the relay, but $\mathbf{\Phi} = \mathbf{I}$) where the destination implements an ML or MMSE receiver matched to the equivalent channel.

In Figure 2, the Symbol Error Rate (SER) is plotted versus ρ_2 with the SNR on the backward channel set to $\rho_1 = 10$ dB. We note that optimized techniques always outperform simple power amplification and the performance gap grows as the number of antennas at the relay increases. Using optimized relaying, diversity gains are achieved by increasing the number of relaying antennas. In contrast, the gains due to additional antennas at the relay are marginal for amplify-only relaying. We finally note that the ML criterion leads to better performance than the MSE criterion. In Figure 3, the SER is plotted versus ρ_2 with the SNR on the backward channel set to $\rho_1 = 20$ dB. Increasing the SNR on the backward channel results in lower error floors as can be observed by comparing with Figure 2. Furthermore, the performance gap between MSE and ML relaying is significantly reduced.

VI. Conclusion

We have considered the problem of relaying spatially-multiplexed signals in a MIMO wireless system. We have proposed two optimal relaying techniques based on the ML and MSE criteria. In both cases, the relay rotates the backward and forward channels in order to align their eigenmodes. The available power is directly poured on the eigenmodes of the equivalent channel according to different allocation policies. We have demonstrated that the proposed relaying techniques achieve error performance improvements, especially when the number of antennas at the relay is increased. In the latter case, our simulation results show that a relay that only amplifies the signal power fails to exploit this additional source of diversity.

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